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Research Article

A Comparative Analysis of Numerical Integration in Explicit and Implicit Methods to be Evaluated in the Linkage Model with Multibody Dynamics Toward an Optimum Design of Support Devices for Human Joint Workload Reduction

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ABSTRACT

In the analysis based on multibody dynamics (MBD), the MBD models are described by ordinary differential equations, and the numerical integration methods applied to the analysis non-negligibly affect the accuracy of the results. In consideration of models containing dynamic components such as spring dampers and flexible elements, the selection of integration methods is highly important not only for accuracy but also computational costs of the analysis, because it easily causes integration errors. For the development of exoskeleton-type assistive devices using flexible elements, an accurate assessment of the effect of their elastic properties is required. In the present study, we applied the implicit integration method in order to improve the accuracy of MBD-based analysis, and numerical errors in computer experiments were significantly reduced in the case of implicit methods with respect to those caused by explicit methods. This verification result can lead to the improvement of the accuracy of the inverse dynamics analysis based on the flexible multibody dynamics (fMBD) theory.

1. Introduction

Multibody dynamics (MBD) has been applied to mechanical systems and biomechanics analysis [1], [2], and it is used to analyze walking robots and musculoskeletal systems [3], [4] with improved analytical stability in numerical analysis [5]. It is becoming the standard practice of kinematic and dynamic analysis for multibody systems which consist of multiple bodies. Therefore, for the displacement analysis of systems including spring-damper elements as dynamic mechanisms, accuracy can be ensured by applying the appropriate numerical integration method [6]. Indeed, considering mechanisms that absorb and utilize reaction force, using flexible materials is one of the interesting options. These components are essential in the detailed analysis of models that reproduce the human joint mechanism and in the development of exoskeleton-type assist devices that support human movements. As an analysis method for the human joint model including elastic elements, there is static analysis in a specific movement posture using a FEM model [7]. However, a detailed analysis of the force and its timing required to reduce joint load through dynamic analysis is necessary for the development of exoskeletal assistive devices. Therefore, it is important to accurately reproduce the mechanisms including these elastic components in computer experiments for the evaluation of human joint models and the assistive device. However, numerical errors can occur in computer analysis when the number of bodies and elastic components increases and the MBD analysis system becomes highly complicated.

The choice of which numerical integration method to apply to the analysis is important in realizing the actual dynamics that occur in the real world, especially for dynamics analysis with deformable bodies. It cannot be solved by simply making the time step size of the integration smaller in the explicit

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numerical method, but by refining the time step adaptively in the implicit numerical integration.

In the current research, we compared and evaluated the outcome of explicit and implicit methods for numerical integration in ordinary differential equations [8], [9] by implementing the implicit methods into the MBD formulation [10]. According to the accuracy evaluation, the advantages of the implicit methods in the computer experiments are clarified, and the suitable selection of the numerical solution for the system in question is realized. It will contribute to a practical numerical solution for flexible multibody dynamics (fMBD), which incorporates the finite element method (FEM) into the original theory of MBD as well as for rigid-body mechanics.

2. Methodology

2.1. Numerical error under MBD analysis

The motions of each mechanical element are described by a generalized coordinate matrix q which consists of x-y coordinates and the body tilt in MBD-based analysis, and the matrix is calculated by second-order integration of the generalized acceleration matrix $\ddot{q}$ which is acquired by solving

Table 1. The components in the differential-algebraic equation.

M	Mass matrix
Φ_q	Jacobian matrix differentiated from constraint matrix in generalized coordinates
$\ddot{q}$	Generalized acceleration matrix
λ	Lagrange multiplier
Q^A	Generalized force
γ	Acceleration equation

the following ordinary differential-algebraic equation (DAE) as Eq. (1). Table 1 shows the significance of the characters in Eq. (1). We solve this DAE by algebraic calculations with the Symbolic Math toolbox in MATLAB.

$$\begin{bmatrix} M & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q^A \\ \gamma \end{bmatrix} \quad (1)$$

In numerical analysis by computer, the generalized coordinate matrix $\boldsymbol{q}$ and the generalized velocity matrix $\dot{\boldsymbol{q}}$ can be obtained by solving this second-order ordinary differential equation with numerical integration methods, and the numerical errors increase cumulatively at each calculation step in numerical integration. In the kinematic analysis, these errors appear as misalignments between joints of the multibody system. The rotational constraints at the system of the knee linkage model as Fig. 1 are defined by the kinematic constraint equations, and it can be kinematically analyzed by solving the DAE so that the constraint equations are always satisfied. When the numerical error in computation accumulates, the nodal coordinates of each element in the system don't satisfy the constraint equations and the deviation of nodal coordinates increases. Therefore, it is possible to suppress the accumulation of errors by applying the numerical integration methods with high accuracy, and the magnitude of the joint coordinate misalignment (Fig. 1) can be improved.

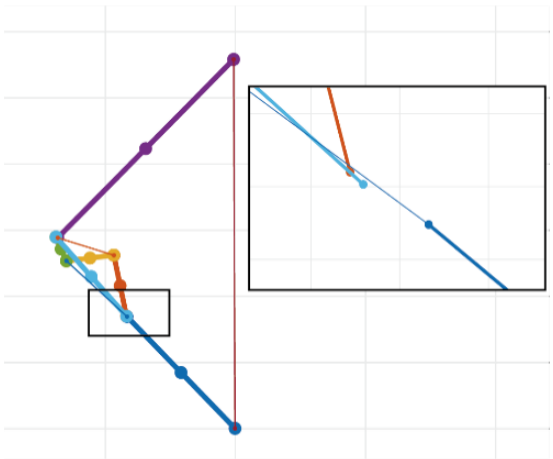
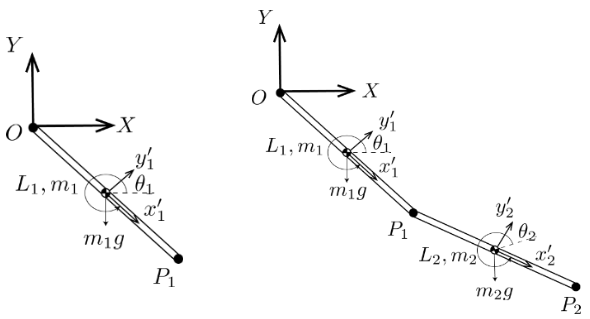


Fig.1 Misalignments in the nodal coordinates in the knee linkage model [6]. It is caused by the accumulation of errors in numerical integration.

For the detailed analysis of the human joint using mathematical models, problems must be addressed as

1) whether it is possible for the complex mathematical model of human joints to decompose in a primitive way for the step-by-step evaluation,
2) how the accuracy of the computational analysis can be evaluated and
3) what is the most crucial method to improve the accuracy in the computational analysis.

As solutions for those problems, we focused on the situation that the accuracy is getting worse when the number of rotary joints is increasing in the first place and decomposed the complex model into sequential joints in the form of pendulum models. In pendulum models, the number of joints can be changed, and the analysis helps estimate the reduction of the accuracy in a comparison associated with the number of joints. A typical reason of why the accuracy is getting worse is the misalignment in the nodal coordinates as shown in Fig. 1, which can be formulated in the criterion to evaluate the accuracy of computational analysis. In the third place, the improvement of the accuracy is mainly relying on the numerical integration method as described in the next section.



a. Single pendulum b. Double pendulum

Fig. 2. Multibody systems of rigid pendulum models.

2.2. *Numerical integration methods*

The explicit and implicit numerical integration methods were applied to the MBD-based dynamic analysis of single and double pendulums (Fig. 2) for the purpose of error comparison and verification in the MBD analysis.

The Runge-Kutta Gill's method [11] was introduced as an explicit method which is an explicit fourth-order Runge-Kutta method, the two-stage fourth-order and the three-stage sixth-order implicit Runge-Kutta (IRK) method, and obtained each numerical error caused by these integration methods. The following equation describes the s-stage IRK method [8], [9]. Table 2 is the Butcher tableau which shows the coefficients *a, b,* and *c* in Eq. (2).

$$\begin{cases} k_i = f(x_0 + c_i h, y_0 + h \sum_{j=1}^{s} a_{ij} k_j) \\ y_{(x_0+h)} = y_0 + h \sum_{i=1}^{s} b_i k_i \end{cases} \tag{2}$$

In MBD analysis, the generalized coordinates matrix q and the generalized velocity matrix $\dot{q}$ are calculated by applying numerical integration to the generalized acceleration matrix $\ddot{q}$ obtained from differential-algebraic equations in Eq. (1). Eq. (4) and Eq. (5) shows the second-order ordinary differential equations for obtaining q and $\dot{q}$ with IRK.

$$\ddot{q} = f(t, q, \dot{q}) \tag{3}$$

$$\begin{cases} k_i = f(t_n + c_i h, q_n + h \sum_{j=1}^{s} a_{ij} l_j, \dot{q}_n + h \sum_{j=1}^{s} a_{ij} k_j) \\ l_i = \dot{q}_n + h \sum_{j=1}^{s} a_{ij} k_j \end{cases} \tag{4}$$

$$(i = 1, \cdots, s)$$

Table 2. The Butcher tableau for IRK

c_1	a_{11}	$\cdots$	a_{1s}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_s	a_{s1}	$\cdots$	a_{ss}
	b_1	$\cdots$	b_s

Table 3. The Butcher tableau for 2-4 IRK

$\frac{1}{2} - \frac{\sqrt{3}}{6}$	$\frac{1}{4}$	$\frac{1}{4} - \frac{\sqrt{3}}{6}$
$\frac{1}{2} + \frac{\sqrt{3}}{6}$	$\frac{1}{4} + \frac{\sqrt{3}}{6}$	$\frac{1}{4}$
	$\frac{1}{2}$	$\frac{1}{2}$

Table 4. The Butcher tableau for 3-6 IRK

$\frac{1}{2} - \frac{\sqrt{15}}{10}$	$\frac{5}{36}$	$\frac{2}{9} + \frac{\sqrt{15}}{15}$	$\frac{5}{36} + \frac{\sqrt{15}}{30}$
$\frac{1}{2}$	$\frac{5}{36} + \frac{\sqrt{15}}{24}$	$\frac{2}{9}$	$\frac{5}{36} - \frac{\sqrt{15}}{24}$
$\frac{1}{2} + \frac{\sqrt{15}}{10}$	$\frac{5}{36} + \frac{\sqrt{15}}{30}$	$\frac{2}{9} + \frac{\sqrt{15}}{15}$	$\frac{5}{36}$
	$\frac{5}{18}$	$\frac{4}{19}$	$\frac{5}{18}$

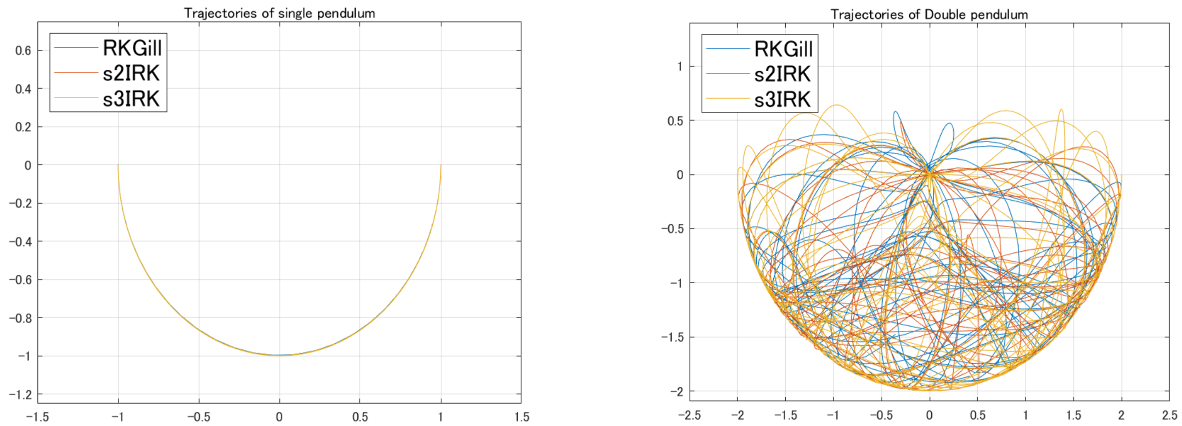
$$
\begin{cases} y'_{n+1} = p_{n+1} = P_n + h \sum_{i=1}^{s} b_i k_i \\ y_{n+1} = y_n + h \sum_{i=1}^{s} b_i l_i \end{cases} \tag{5}
$$

The explicit and implicit numerical integration methods were applied to the MBD-based dynamic analysis of single and double pendulums (Fig. 2) for the purpose of error comparison and verification in the MBD analysis. The coefficients k_i and l_i in Eq. (4) are derived by the Newton-Raphson method in each numerical integration step, and q and $\dot{q}$ are solved by Eq. (5) with these coefficients. Table 3 and Table 4 show the Butcher tableau applied to Eq. (5) for 2stage 4order IRK and 3stage 6order IRK respectively [8].

3. Results and Discussion

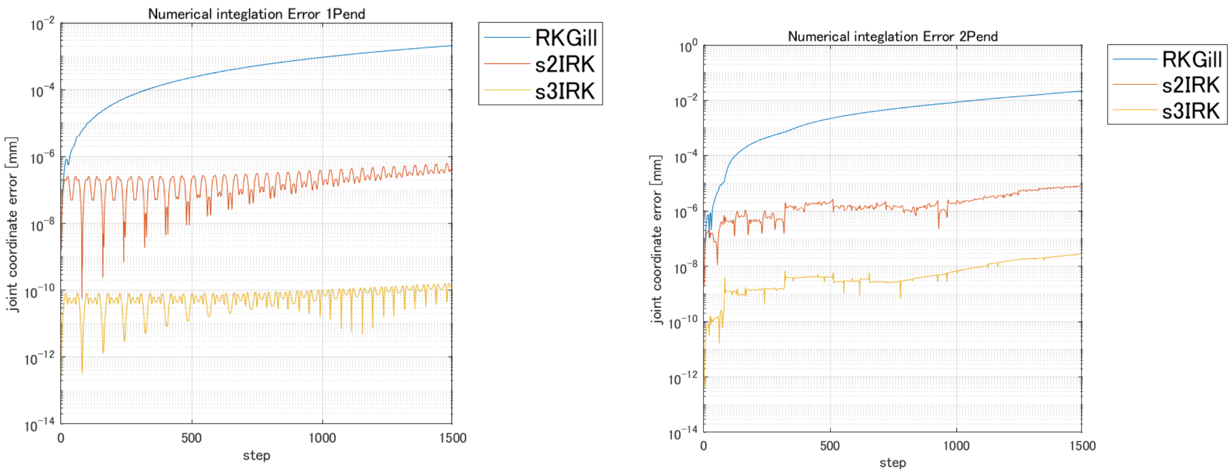
3.1. *Accuracy verification of numerical integration*

The numerical errors accumulated during the calculations were verified with MBD-based analysis of single and double pendulum models which is implemented in each numerical integration method. Here,



a. Single pendulum b. Double pendulum

Fig.3 Trajectories drawn by the pendulum endpoints



a. Single pendulum b. Double pendulum

Fig. 4. Comparison of joint node misalignments generated by each numerical integration method

the time increment in each calculation step is $h = 0.1$. Fig. 3 shows the differences in the trajectories of the endpoint caused by each integration method.

The transitions of the magnitude of the misalignments in the joint coordinates at each step of the numerical integration calculation are shown in Fig. 4.

According to these figures, the results of the trajectory analysis of the double pendulum were changed by the adapted numerical integration methods, and the magnitude of the joint node misalignments with RK-Gill is significantly increased compared to other methods. It is revealed that the implicit method makes the errors smaller than the explicit method in both single and double pendulum analysis. Furthermore, it is obvious that the higher-order solution generates smaller numerical errors. Thus, in the analysis of an MBD-based system with multiple elements, the accuracy of numerical integral calculations can be evaluated from the errors which appear as the joint coordinate misalignments of the mechanical elements consisting of the multibody system to be analyzed.

3.2. *MBD analysis performance depends on the numerical integration*

In dynamic analysis with numerical calculations, the analysis time is also valued as same as analysis accuracy. The increase in numerical computation time because of the increase in the number of bodies consisting of the multibody system and the expansion of the order of the implicit integration method is shown in Fig. 5 In this section, the number of repetitive calculation steps is 1500.

When the analytical system has more body and variables, the analysis times increase greatly with the enlargement of implicit numerical integration order. In the implicit method, coefficients in simultaneous equations (Eq. 4) are solved by the Newton-Raphson method in each integration step, and numerical substitutions into the symbolic matrix are performed to solve the simultaneous equations. These processes increase the computation time. In addition, increasing the number of implicit stages enlarges the size of the symbolic matrix and takes more computation time. Therefore, incorporating an efficient numerical substitution process for symbolic matrices enables the implementation of higher-order implicit methods in MBD-based analysis.

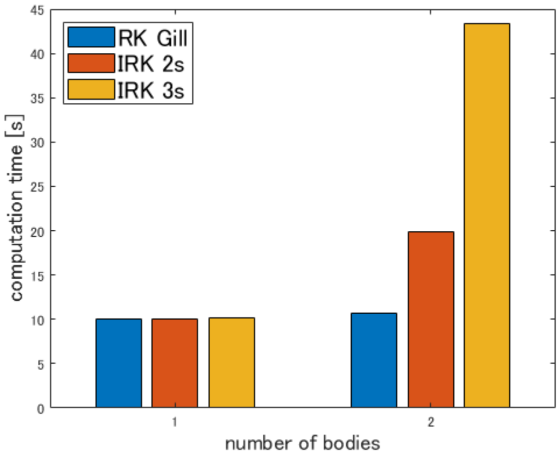


Fig. 5. The calculation times of numerical integrations

4. Conclusion

In the present study, as a primitive step to analyze mathematical models of human joints, we assumed pendulum models to be a simplification to decompose the complex model and proposed the accuracy estimation method in the form of the joint coordinate error. In the numerical integration based on the explicit method with the Runge-Kutta Gill, the error was clearly increased (Fig. 4) apparently rather than implicit methods as s2IRK and s3IRK. In s2IRK and s3IRK, individual accuracies were getting worse from 10^{-6} and 10^{-10} in the single pendulum to 10^{-5} and 10^{-8} in the dual pendulum model. It means that the error is getting worse in the range $[10^{-2}, 10^{-1}]$ for increasing a rotary joint. As shown in Fig. 1, the knee joint model has four rotary joints in the central part, and therefore the result implies that joint coordinate error will be to $[10^{-3}, 1]$ [mm] and $[10^{-7}, 10^{-4}]$ [mm] in s2IRK and s3IRK. It indicates that the selection of numerical integration methods is highly important for analyses of kinematic and dynamic properties when MBD-based models were used.

In further analysis, a consistent verification will be done in the dynamics/inverse dynamics analysis of the human knee joint mechanism with flexible elements, and then the model analysis requires to use fMBD in various conditions such as walking, jumping, standing, and sitting. The standardization to obtain assured results is expected and dynamical analysis of exoskeleton-type assistive devices can easily obtain by adopting an optimum numerical integration method.

On the other hand, for the fundamental improvement of the performance of the numerical methods in senses of accuracy and computational costs, adequate consideration for embedding symbolic matrix formulas in codes of MATLAB is an inevitable mission towards further analysis for the higher order IRK caused by the complexity of model structure with various mechanisms.

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