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Research Article

Improvement of the Database-Driven PID Control Method by H-Infinity Norm of the Local Model

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ABSTRACT
PID controllers are used in many applications and are considered an example of a fixed structure controller. For each specific system, the controller's PID parameters must be adjusted accordingly. Therefore, database-driven control, which automatically adjusts PID controller parameters using a large amount of experimental data, is attracting attention. Database-driven control is one of the methods that does not require model information. Therefore, in database-driven PID (DD-PID), the Jacobian of the system used for the steepest descent is unknown. In this paper, we propose an optimization method using gradient descent based on a local model of the system and confirm its effectiveness through simulation using the polystyrene reactor model as an example.

1. Introduction

PID controllers are widely used in industry [1] and are one of the typical examples of controllers with a fixed structure. To adjust the parameters appropriately, it is effective to identify a mathematical model of the system, and many methods have been reported so far [2]. However, the systems in general use have various nonlinearities, making it difficult to construct a detailed model, and the desired control performance may not be achieved with a conventional fixed PID controller. Therefore, database-driven control has been actively studied in which PID controller parameters are automatically adjusted according to system characteristics using a large amount of experimental data [3], [4], [5], [6].

The DD-PID method, one of the database-driven control methods, constructs a local model based on data accumulated in a database and updates the PID gains using a nonlinear optimization method. This local model is computed by extracting data from the database that closely resembles the query generated using the current operational data as neighborhood data. Design of the database and query is important to construct a local model using the JIT method, and they also affect the control results. Next, correct the gain obtained using the JIT method. A nonlinear optimization method is used to modify the gain. In this paper, we use the gradient descent method. To be effective with the gradient descent method, it is important to set the appropriate coefficients. If appropriate learning coefficients are not given, there is a problem that PID gains are calculated, resulting in unstable control results.

In this paper, we examine a database design method for the DD-PID method. It is which identifies the local models of the system using the Recursive Least Squares (RLS) method. And estimate to H_∞ norm. It uses the estimated H_∞ norm to design the database. Furthermore, the Jacobian of the local model identified by RLS is reflected in the gradient method to optimize the PID gain update. Finally, the proposed method is validated by numerical simulations using the polystyrene reactor model.

Table 1 shows a list of variables used in this paper.

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1.1. *Controller*

In practical control scenes, achieving smooth response characteristics is desirable. In this paper, we design the controller using an I-PD control scheme.

Table 1. Setting parameters for DD-PID

Specification	Symbol
α - th epoch	α
Number of samples of epoch	M
Time	$t_{(\alpha)}$
Sampling time	T
Jacket temperature	$y(t)$
Reaction temperature	$u(t)$
Order of the $y(t)$ and the $u(t)$	N_y, N_u
Delay operator	z
Database	$\Omega(\alpha)$
Querry	$q(t)$
Degree of similarity	$S(\boldsymbol{q}(t), \boldsymbol{q}(\tau))$
Threshold of degree of similarity	κ
Neighborhood data	$\Gamma(t)$
Evaluation function	$L(\alpha)$
Acceptable error range of $L(\alpha)$	ρ_L
PID gains	$\boldsymbol{\theta} = [K_P, K_I, K_D]$
Learning vector for gradient descent	$\boldsymbol{\eta} = [\eta_P, \eta_I, \eta_D]^T$
Controller model	$C(\boldsymbol{\theta})$
Plant model	$H(z)$
Model parameter of the control target	a_1, a_2, b_1, b_2
White noise	$\zeta(t)$
Reference model	$G_m(z)$
Number of the Query registered in the database	$n(\cdot)$

This paper utilizes the I-PD controller since achieving smooth response characteristics is desirable in practical control applications [7]. Eq. (1) shows the I-PD controller.

$$u(t) = u(t-1) + K_I(t)e(t) - K_P(t)\Delta y(t) - K_D(t)\Delta^2 y(t) \quad (1)$$

Here, $K_P(t)$,$K_I(t)$, and $K_D(t)$ represent the proportional, integral, and derivative gains at each time instant. And z is a delay operator that satisfies $z^{-1}y(t) = y(t-1)$, where $\Delta = 1 - z^{-1}$. $e(t)$ denotes the deviation, defined by Eq. (2).

$$e(t) = r(t) - y(t) \quad (2)$$

1.2. *Reference model*

The reference model for the DD-PID method is defined by Eq. (3), (4) and (5). Here μ is a parameter related to attenuation, ρ is a parameter related to rising time, and T is the sampling time [8].

$$G_m(z) = \frac{P(1)z^{-1}}{P(z)} = \frac{(1 + p_1 + p_2)z^{-1}}{1 + p_1 z^{-1} + p_2 z^{-2}} \quad (3)$$

$$y_m(t) = G_m(z)r(t) \quad (4)$$

$$\begin{cases} p_1 = -2e^{-\frac{\rho}{2\mu}} \cos\left(\frac{\sqrt{4\mu - 1}}{2\mu}\rho\right) \\ p_2 = e^{-\frac{\rho}{\mu}} \\ \mu = 0.25(1 - \delta) + 0.51\delta \\ \rho = \frac{T}{\sigma} \end{cases} \quad (5)$$

1.3. *Proposed method*

The DD-PID method proposed in this paper updates the PID gains by the following steps.

Step.1 Setting of the reference model and experiment using initial PID Gains

The DD-PID method cannot update parameters when there is no data in the database. Therefore, an initial database is constructed using a stable controller. The stable controller is assumed to be pre-constructed using the CHR method or other methods. The database used in this paper is defined by Eq. (6). In addition, $\|G(t)\|_\infty$ is the H_∞ norm of the local model calculated using the RLS method, $\boldsymbol{\phi}(t_{(\alpha)})$ is the information vector, and $\boldsymbol{\theta}(t_{(\alpha)})$ is the gain vector of the controller.

Note that the H_∞ norm coincides with the maximum gain of the signal amplified by the transfer function $G(t)$.

Here, $t_{(\alpha)}$ represents the time of the α-th epoch.

$$\boldsymbol{\Omega}(\alpha) = \left[\boldsymbol{\phi}^T(t_{(\alpha)}), \|G(t_{(\alpha)})\|_\infty, \boldsymbol{\theta}^T(t_{(\alpha)})\right]^T \quad (6)$$

The control target model to be estimated by RLS is defined by Eq. (7).

$$H(z) = \frac{b_1(t)z^{-1} + b_2(t)z^{-2}}{1 + a_1(t)z^{-1} + a_2(t)z^{-2}} \quad (7)$$

The control target assumed in this paper is assumed to have no direct derivatives and the order of the system does not vary with time. The reason for using a quadratic delay system here is that identification by a quadratic delay system can represent most mechanical systems.

$G(z)$ is given by Eq. (8).

$$G(t) = \frac{H(z)C(\boldsymbol{\theta})}{1 + H(z)C(\boldsymbol{\theta})} \quad (8)$$

Define $\boldsymbol{\phi}(t_{(\alpha)})$ and $\boldsymbol{\theta}(t_{(\alpha)})$ by Eq. (9) and (10).

$$\boldsymbol{\phi}(t_{(\alpha)}) = \left[y(t_{(\alpha)}), u(t_{(\alpha)})\right]^T \quad (9)$$

$$\boldsymbol{\theta}(t_{(\alpha)}) = [K_P(t_{(\alpha)}), K_I(t_{(\alpha)}), K_D(t_{(\alpha)})]^T \quad (10)$$

Step. 2 Getting queries and neighborhood data

Obtain the query $q(t)$ while operating the control target. $q(t)$ is defined by Eq. (11). Here, N_y and N_u are the orders of $y(t)$ and $u(t)$.Here, N_y and N_u are constants that are determined by the control designer by trial and error.

$$\boldsymbol{q}(t) = [y(t), \cdots, y(t - N_y + 1), u(t), \cdots, u(t - N_u + 1), \|G(t)\|_\infty]^T \quad (11)$$

In this paper, control is performed using the PID gain $\boldsymbol{\theta}(t(\alpha - 1))$ at $t(\alpha - 1)$.

Using the retrieved query, the proximity distance of the data to the information vector in the database is calculated, and neighboring data are selected. Although several algorithms have been proposed for selecting nearby data from a large amount of data, this paper uses Kinoshita's [8] method for obtaining nearby data based on similarity. Kinoshita's method uses statistical methods and is superior to the K-NN method in that it is computationally less expensive. The similarity between the information vector and the query is given by Eq. (12).

$$S(\boldsymbol{q}(t), \boldsymbol{q}(\tau)) = \prod_{j=1}^{N_y+N_u+1} \frac{1}{\sqrt{2\pi h_j^2}} \exp\left\{-\frac{(q_j(t) - q_j(\tau))^2}{2h_j^2}\right\} \quad (12)$$

h_j is the bandwidth of the probability density function, $q_j(t)$ is the j-th element of the query at time t.σ_j and h_j is given by Eq. (13) and (14).

$$\sigma_j = \sqrt{\frac{1}{n(\Omega)} \sum_{k=1}^{n(\Omega)} (q_j(k) - \mu_j)^2} \quad (13)$$

$$h_j = 1.06\sigma_j n(\Omega)^{-\frac{1}{5}} \quad (14)$$

This method of estimating the kernel density function is called Plug-In method.

The selection of neighboring data is based on similarity according to Eq. (15). The designer sets $\kappa(0 \le \kappa \le 1)$.

$$S(\boldsymbol{q}(t), \boldsymbol{q}(\tau)) \ge \kappa \prod_{j=1}^{N_y+N_u+1} \frac{1}{\sqrt{2\pi h_j^2}} \quad (15)$$

The neighboring data selected by Eq. (15) are represented by $\Gamma(t)$.

Step. 3 Local model calculation by JIT method

Using the neighborhood data selected in Step.2, construct a local model using Eq. (16) and (17) to obtain the PID gain $\tilde{\boldsymbol{\theta}}(t)$.

$$\tilde{\boldsymbol{\theta}}(t) = \sum_{i=1}^{n(\Gamma(t))} \boldsymbol{\omega}_i \boldsymbol{\theta}(i), \qquad \sum_{i=1}^{n(\Gamma(t))} \boldsymbol{\omega}_i = 1 \quad (16)$$

$$\boldsymbol{\omega}_i = \frac{S(\boldsymbol{q}(t), \boldsymbol{q}(i))}{\sum_{j=1}^{n(\Gamma(t))} S(\boldsymbol{q}(t), \boldsymbol{q}(j))} \quad (17)$$

The gain $\tilde{\boldsymbol{\theta}}(t)$ calculated by the JIT method may be inappropriate. Therefore, the gain $\tilde{\boldsymbol{\theta}}(t)$ calculated in step 3 is corrected using the gradient method. The gain correction by the gradient method is given by Eq. (18). As shown in Eq. (18), the gain correction by the DD-PID method is delayed by one step. This is because the data used to update the gain by the gradient method is related to $u(t)$.

$$\boldsymbol{\theta}(t) = \tilde{\boldsymbol{\theta}}(t) - \boldsymbol{\eta} \frac{\partial J(t+1)}{\partial \tilde{\boldsymbol{\theta}}(t)} \quad (18)$$

Where $\boldsymbol{\eta}$ is the learning coefficient for the gradient method, $J(t+1)$ is the evaluation function, and $\varepsilon(t)$ is the difference between the system output and the reference model output. From these relationships, the partial derivative in Eq. (18) can be expanded using the differential chain rule as in Eq. (19).

$$\frac{\partial J(t+1)}{\partial \tilde{\boldsymbol{\theta}}(t)} = \frac{\partial J(t+1)}{\partial \varepsilon(t+1)} \frac{\partial \varepsilon(t+1)}{\partial y(t+1)} \frac{\partial y(t+1)}{\partial u(t)} \frac{\partial u(t)}{\partial \tilde{\boldsymbol{\theta}}(t)} \quad (19)$$

Here, $J(t)$ and $\varepsilon(t)$are defined by Eq. (20).

$$J(t) = \frac{1}{2}\varepsilon(t)^2 = \frac{1}{2}\{y(t) - y_m(t)\}^2 \quad (20)$$

The Jacobian of the system is given by Eq. (21) from Eq. (7).

$$\frac{\partial y(t+1)}{\partial u(t)} = b_1(t+1) \quad (21)$$

The differential chain rule can be calculated using Eq. (21) and (22).

$$\begin{cases} K_P = \tilde{K}_P + b_1(t+1)\eta_P \varepsilon(t+1)\Delta y(t+1) \\ K_I = \tilde{K}_I - b_1(t+1)\eta_I \varepsilon(t+1) e(t+1) \\ K_D = \tilde{K}_D + b_1(t+1)\eta_D \varepsilon(t+1)\Delta^2 y(t+1) \end{cases} \quad (22)$$

Step. 4 Update the database

To store the controlled variable, manipulated variables, and system parameters obtained from experiments in the database.

Step. 5 Evaluation and Convergence Decision

Update the controller parameters by repeating Step 2 through Step 5 until the evaluation function shown in Eq. (23) and (24) becomes sufficiently small or is repeated a specified number of times.

$$L(\alpha) = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\{y_m(t_i) - y(t_{(\alpha),i})\}^2} \tag{23}$$

$$|L(\alpha) - L(\alpha-1)| \le \rho_L \tag{24}$$

ρ_L is a convergence threshold, which is determined by the user by trial and error.

2. Simulation

2.1. *System parameter settings*

In this paper, the effectiveness of the proposed method is verified by numerical simulations using the polystyrene reactor model shown in Eq. (25) [3], [6], [9].

$$y(t) = 0.804y(t-1) + 5.739 \times 10^{15} \cdot e^{-\frac{E_a}{R(y(t-1)+273)}} + 0.148u(t-1) + \xi(t) \tag{25}$$

Here,$u(t)$is the jacket temperature, $y(t)$ is the reaction temperature, $E_a = 240$, $R = 0.01986$ and $\xi(t)$ is gaussian white noise. In this paper, temperature is the Celsius degree. This system has ability to be strongly non-linear at about 75 Celsius degrees or more.

Target temperature was set as in Eq. (26).

$$r(t) = \begin{cases} 60(& 0 < t \le 100) \\ 70(100 < t \le 200) \\ 85(200 < t \le 300) \end{cases} \tag{26}$$

The reference model is designed as in Eq. (27). Here, $\delta = 0$ and $\sigma = 5$.

$$P(z) = 1 - 1.34z^{-1} + 0.449z^{-2} \tag{27}$$

The initial database is constructed using the initial PID gain shown in Eq. (28).

$$K_P = 9.0, K_I = 0.5, K_D = 1.0 \tag{28}$$

Other parameters are shown in Table 2.

Fig. 1 shows the results of control using Eq. (28). From Fig. 1, the ability to follow the output signal of the reference model is poor.

Table 2. Setting parameters for DD-PID

Sample time	$T = 1[s]$
Order of the $y(t)$ and the $u(t)$	$N_y = 5$ $N_u = 5$
The constant value for the RLS	$a = 1000$ $\boldsymbol{P}_0 = \alpha\boldsymbol{I}$ $\boldsymbol{\zeta}_0 = \boldsymbol{O}$
Value of the forgetting coefficient	$\rho_F = 0.99$
Similarity threshold	$\kappa = 0.85$
Gradient dissent coefficient	$\eta_P = 13 \times 10^{-3}$ $\eta_I = 13 \times 10^{-3}$ $\eta_D = 8 \times 10^{-3}$
Convergence value threshold	$\rho_L = 10^{-4}$

2.2. *Simulation results*

In this paper, we setting to the parameter are $\alpha_1 = 0.5, \alpha_2 = 0.1, N(0) = 6$, and the gradient descent gains setting to (1) $\boldsymbol{\eta}_{(1)} = [2 \times 10^{-4}, 2 \times 10^{-4}, 2 \times 10^{-4}]^T$, (2) $\boldsymbol{\eta}_{(2)} = [4 \times 10^{-4}, 4 \times 10^{-4}, 2 \times 10^{-4}]^T$, (3) $\boldsymbol{\eta}_{(3)} = [6 \times 10^{-4}, 6 \times 10^{-4}, 2 \times 10^{-4}]^T$ and (4) $\boldsymbol{\eta}_{(4)} = [8 \times 10^{-4}, 8 \times 10^{-4}, 2 \times 10^{-4}]^T$.

Fig. 2 shows result using the Yamamoto's DD-PID update method for polystyrene reactor model [8], here Fig. 2-a shows the results of the control, Fig. 2-b shows tuning PID Gains, and Fig. 2-c shows the evaluation function values. The evaluation function's value is $L(34) = 1.396$ using $\boldsymbol{\eta}_1$, $L(137) = 1.708$ using $\boldsymbol{\eta}_2$, and $L(46) = 1.449$ using $\boldsymbol{\eta}_3$.Here, when the evaluation function value using $\boldsymbol{\eta}_4$, it is diverged. The lowest values for the Yamamoto's method are $L(33) = 1.396$ using $\boldsymbol{\eta}_{(1)}$. The PID Gains when $\boldsymbol{\eta}_{(3)}$ is inappropriate because the $K_D(t)$ Gains are negative.

Fig. 3 shows result using the proposed method for polystyrene reactor model [8], here Fig. 3-a shows the results of the control, Fig. 3-b shows tuning PID Gains, Fig. 3-c shows the evaluation function values, Fig. 3-d shows result of the H_∞ norm, and Fig. 3-e shows the result of the $b_1(t)$.

From Fig. 3-a, its response has improved compared to Fig. 2. In particular, the ability to follow the reference model in the rising section has been improved.

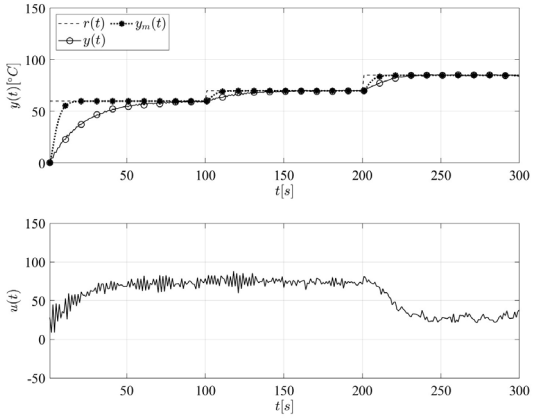
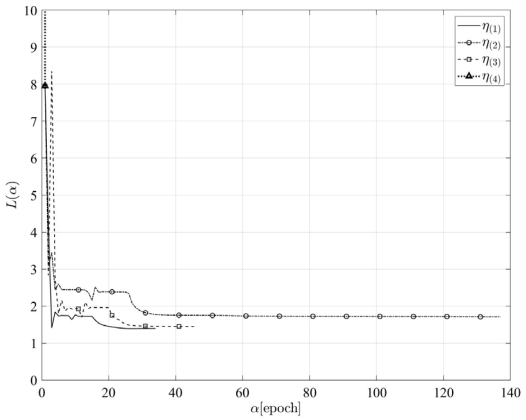
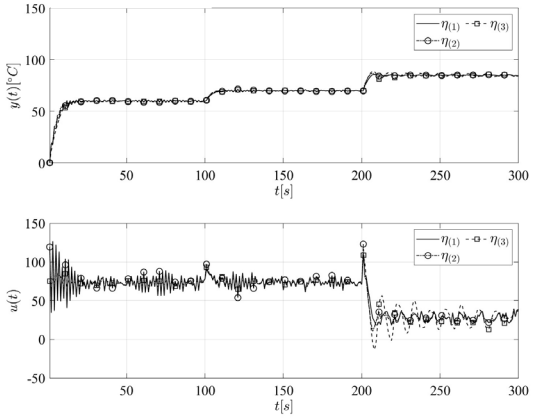


Fig.1 Result of $y(t)$ and $u(t)$ using the initial PID Gains

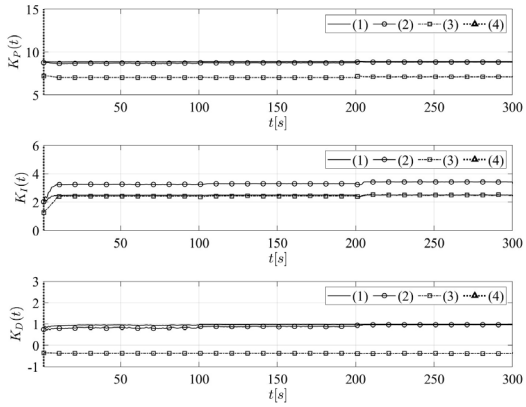


(c) Results of the evaluation function $L(\alpha)$

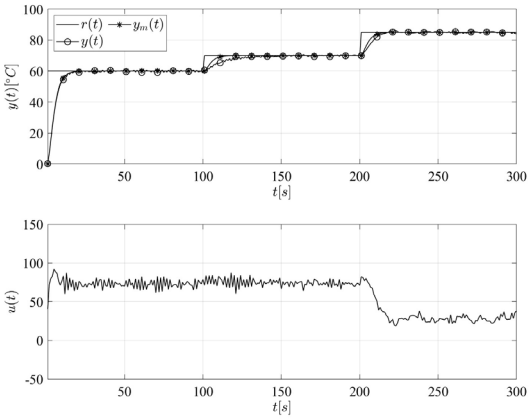
Fig.2 a, b, c results of the Yamamoto's method using learning costs of $\eta_{(1)}, \eta_{(2)}, \eta_{(3)}$and $\eta_{(4)}$ [3].



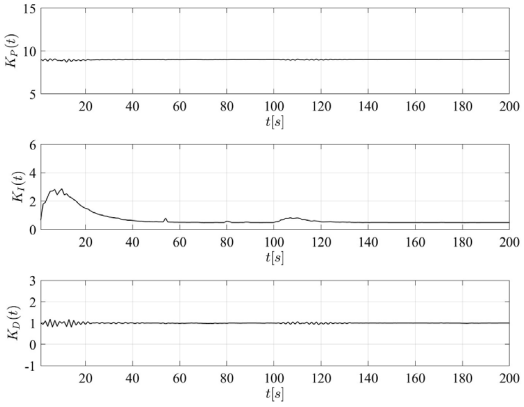
(a) Result of the $y(t)$ and $u(t)$ using $\eta_{(1)}$



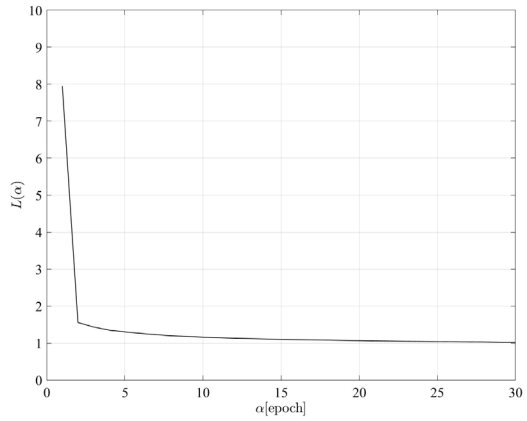
(b) Result of the PID Gains



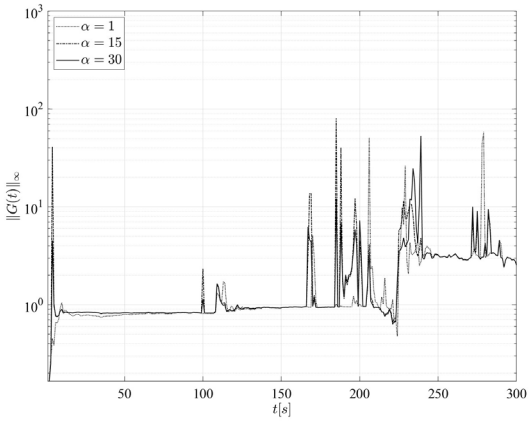
(a) Result of the $y(t)$ and $u(t)$



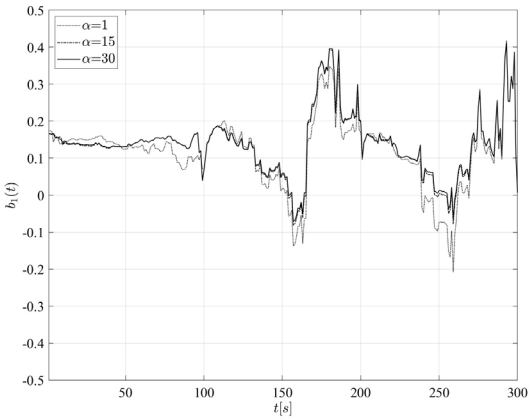
(b) Result of the PID Gains



(c) Result of the evaluation function $L(\alpha)$



(d) Result of the H-Infinity norm of the system



(e) Result of the $b_1(t)$

Fig.3-a, b, c, d, e Results using the proposed method

The PID gain changes more significantly compared to Yamamoto's method. From Fig. 3-c, the evaluation function for the proposed method is $L(30) = 1.02$. The evaluation function value and epoch number are better than Yamamoto's method. From Fig. 3-d, the H_∞ changes significantly at $t = 160$. This is thought to be because the nonlinearity suddenly becomes stronger around $t = 160$.From Fig. 3-e, the graph is positive in most areas. The Jacobian of this system is positive as shown in Eq. (25). The negative portion is the effect of estimation error by the RLS.

3. Conclusion

This paper proposed the gradient descent using local model parameters for DD-PID. And the obtained H_∞ of the system was reflected in the database. The effectiveness of the proposed method was confirmed by numerical simulation by adapting it to the polystyrene reactor model discussed in Yamamoto's studies. The Yamamoto's update method becomes $L(33) = 1.396$. The simulation results take time to converge for the Yamamoto's update method, and $L(30) = 1.02$ for the proposed method, and good results were obtained. And the proposed method's evaluation function value is more than the Yamamoto's method. One of the reasons for such good results may be that the system Jacobian was given, which enabled learning with more correct coefficients. A side effect of giving the system Jacobian is to allow for a larger learning coefficient. This is thought to result in faster PID gain updates. These results suggest that the proposed method is effective for better PID gain updating.

Future research is needed to introduce the proposed algorithm into actual equipment. For this purpose, it is necessary to consider the DD-PID method, which can directly specify the design of stability. In addition, database design methods need to be discussed.

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Authors Introduction

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