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Research Article

An analysis of Quoridor by reusing results from the reduced version

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ABSTRACT

Retrograde analysis is a representative game analysis method that initially lists all legal positions and searches for the best move by analyzing from the final positions to the initial position. This method is effective for analyzing the games where the same position appears multiple times during sequential analysis from the initial position. However, it also has the drawback of requiring a huge amount of memory to enumerate the legal solutions. In this paper, we focus on the board game "Quoridor" released by Gigamic.

1. Introduction

A game tree is a graph structure that represents a game as a directed graph with game positions as nodes and players' moves as edges [1]. The standard method for game analysis is to expand this game tree. Fig. 1 illustrates a game tree for tic-tac-toe, displaying all moves up to the second move. A complete game tree enables the identification of the best move. However, there are some problems with this method. One of the problems is the possibility of repeating the same positions. In Tic-Tac-Toe, the number of squares available for placement decreases with each move to an arbitrary position, and positions that have appeared before the current position will never appear again.

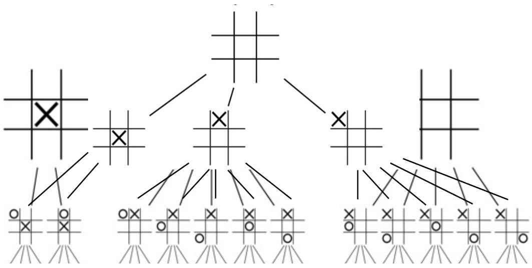

Fig. 1 The game tree of Tic-tac-toe

However, in chess and shogi, once a move is made, it can be reverted to the previous position, allowing the same position to appear multiple times at different nodes of the game tree.

Retrograde analysis was developed to address the complexities of game analysis. This method involves examining all regal positions and tracking win/loss outcomes from the final position back to the initial one. Once the win-loss outcomes stabilize, the initial position is categorized as a must-win game, a must-lose game, or a tie for the first player. Retrograde analysis effectively prevents loops in the game tree that may occur during

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gameplay. However, it necessitates examining and analyzing all legal positions, leading to memory-related issues. Consequently, efforts are made to minimize the number of examined positions as much as possible.

This study aims to reduce the number of positions in retrograde analysis by reusing the results from a reduced version.

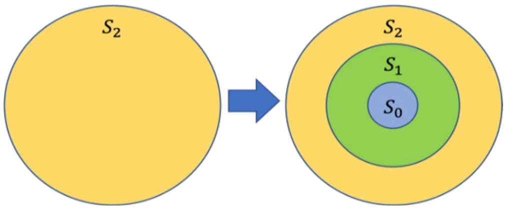

Fig. 2 Before and after reusing.

In this study, the phase set is organized according to Fig. 2. When the phases follow the sequence S_k(k=0,1,...) such that $S_0 \subset S_1 \subset S_2$..., with phase transitions occurring only between adjacent phases in S_k, the results analyzed in S_k(k=0,1,...,n-1) can be reused. This decreases the number of phases needed to be enumerated in S_n. It's important to note that S_{k-1} represents a simplified version of S_k, where the number of items is reduced by one step. Here, the reduced version of the original game refers to a scenario where the number of items or board size has been reduced.

2. Quoridor

In this paper, we discuss the miniature version of the board game Quoridor [2]. This section outlines the rules for a 5x5 board designed for two players, each with one fence.

2.1. *Object of the Game*

The objective of the game is the same as the standard version [3] : to be the first to reach the line opposite one's baseline.

2.2. *Game Play (2 players)*

Each player takes turns choosing to move their pawn or to place a fence. A player who has run out of fences must move their pawn. At the start of the game, the board is empty. Place your pawn in the center of the first row on your side of the board. Your opponent takes another pawn and places it in the center of the first row on their side of the board. Then, each player takes one fence.

2.3. *Pawn moves*

As shown in Fig. 3, the pawns are moved one square at a time, either horizontally or vertically, forwards or backwards, but never diagonally. The pawns must navigate around the fences. If, while you move, you encounter your opponent's pawn, you can jump over it.

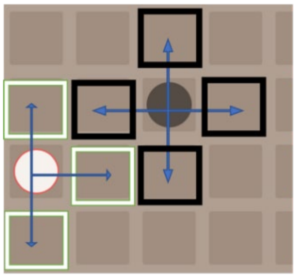

Fig. 3 How to move pawn

The white squares indicate where the white pawn can move, while the black squares indicate where the black pawn can move.

2.4. *Positioning of the fences*

The fences must be placed between two sets of two squares. By strategically placing fences, you can compel your opponent to navigate around them, thereby increasing the number of moves they must make. However, it is not permissible to block your opponent's pawn completely; it must always have a clear path to its goal, with at least one square available for movement.

2.5. *Jumping over opponent's pawn*

As shown in Fig. 4, if two pawns are adjacent and there's no fence between them, the current player can jump over the opponent's pawn, moving ahead an extra square. However, if there's a fence behind the opponent's pawn, the player can move their pawn to the left or right of the opponent's pawn instead.

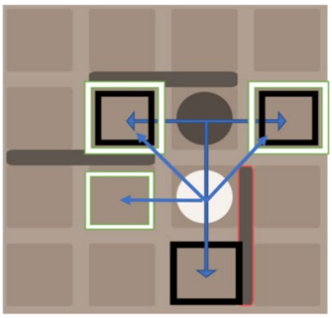

Fig. 4 Encounters Between Pawns and Fences

2.6. *End of the game*

The winner is the first player to reach one of the five squares opposite their baseline.

3. Retrograde Analysis

In this study, we conducted an experiment using retrograde analysis [4], [5]. This method involves retracing steps from the final stage, where victory or defeat is determined, back to the initial board, one step at a time. During this process, if the preceding move leads to a victory, that information is recorded; likewise, if all moves lead to defeat, that information is recorded, and the process repeats until the outcome of the initial phase is determined. The update of win/loss information is depicted in Fig. 5. Blue represents an undecided game, white represents a game won by the white player, and black represents a game won by the black player.

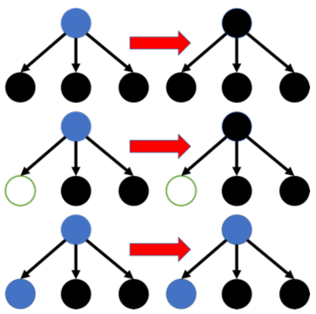

Fig. 5 The updating of win/loss information

One advantage of this method is its ability to consider tie scenarios, wherein players repeat the same moves, referred to as "Sennichite."

4. Research Methods

This study proposes a method to determine the outcome of a game by reusing results obtained from a reduced version of the game board. The term "reduced version" refers to adjustments made to the game board's size or the number of items. For example, in a fixed-size Quoridor game, reuse occurs in situations where irreversible moves are made, typically related to placing fences. In a set of game states Sn where the number of fences on the board is n or less, a containment relationship $S_0 \subset S_1 \subset S_2 \ldots$ holds. Thus, defining $S'_k = S_k \setminus S_{k-1}$ ($k=1,2,3, \ldots$), we have

$$S_k = S_{k-1} \cup S'_k,\quad S_{k-1} \cap S'_k = \varphi.$$

Consequently, during the analysis of game state S_k, it suffices to examine whether S'_k introduces updates to the values of S'_{k-1}, given that the analysis of state Sk-1 has already been completed. If updates occur, they are propagated sequentially to S'_{k-2}, S'_{k-3}, and so on.

This method allows for the analysis of newly added game states and the propagation of information to the reduced version, making it sufficient to enumerate only S'_n when analyzing S_n.

5. Results

Initially, in the case where both players have one fence each on a 5x5 board (S_2), we enumerated 498,764 positions including S0 and conducted retrograde analysis directly. As a result, it was confirmed that being the second player ensures victory [6].

Table 1 Number of Game States and Changes in Win/Loss Information

Number of fences installed	Number of all phases	Number of phases in which win/loss information changed
0 fence	960	
1 fence	61440	764
2 fence	436364	49273

Table 1 shows the number of enumerated positions and the number of positions where win/loss information changed when applying the proposed method to split the set of positions into subsets based on the number of fences on the board (0 fences for S_0, 1 fence for S'_1, and 2 fences for S'_2) during retrograde analysis of S_2. For example, enumerating 436,364 positions in S'_2 and propagating them to S'_1 through retrograde analysis resulted in changes in win/loss information for 49,273 positions. This means that the number of positions enumerated at once has decreased by approximately 14%. Thus, an improvement in space complexity was observed. However, concerning the Quoridor game addressed in this study, the impact of changes in win/loss information was about 80%, which was significant, and did not lead to a reduction in computational complexity.

6. Conclusion

Retrograde analysis is an effective method for analyzing the outcomes of games that allow for draws, but it presents challenges in terms of space complexity rather than time complexity. Therefore, retrograde analysis is difficult to apply to problems other than those with a small number of legal positions.

In this study, we aimed to reduce the number of enumerated legal positions in game analysis using retrograde analysis, and proposed a method to partition the set of legal positions by irreversible moves. As a result, the effectiveness of the proposed method was confirmed.

However, when dividing the set of legal positions, it is necessary to minimize the impact of newly added positions on the previously analyzed ones. This can lead to significant savings in terms of time complexity. It should be possible to reduce the computational complexity requiring reanalysis by devising partitioning methods, even in the Quoridor game studied in this research.

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