Journal of Robotics, Networking and Artificial Life
Vol. 11(2); December (2025), pp. 134-138
ISSN (Online): 2352-6386; ISSN (Print): 2405-9021
https://alife-robotics.org/jrnal.html

Research Article

Kinematics and Simulation for Mobil Robot Adapted Three Omni Rollers

Kenji Kimura[1], Kazuki Nakayama[1], Katsuaki Suzuki[2*], Kazuo Ishii[3*]
[1]*Department of Control Engineering, National Institute of Technology, Matsue College, 14-4 Nishi-ikuma-cho, Matsue-shi, Shimane,690-8518, Japan*
[2]*Kumamoto Industrial Research Institute, 3-11-38 Higashi machi, Higashi-ku, Kumamoto 862-0901, Kumamoto, Japan*
[3]*Faculty of Engineering, University of Miyazaki, 1-1 Gakuen Kibanadai-NishiMiyazaki, 889-2192, Japan*
Email: k-kimura@matsue-ct.jp, k-suzuki@kumamoto-iri.jp, ishii@brain.kyutech.ac.jp
[*]Corresponding Author

ARTICLE INFO

Article History

Received 10 November 2024
Accepted 31 July 2025

Keywords

Trajectory analysis
Mobile robot
Angular velocity

ABSTRACT

Mobile robots are expected to play an active role in the logistics industry, and their development is underway. There are many examples of mechanisms with three omni-rollers as omnidirectional mobile mechanisms, and methods for analyzing and verifying their kinematics have been proposed. In this study, we derive a robot trajectory equation for roller drive. In addition, we verified the kinematics in a simulation environment on a PC, with the goal of reducing costs and time in a simple verification experiment with good visibility.

1. Introduction

In recent years, in fields such as logistics, speed and time-efficient mobility are required for mobile robots. Therefore, the wheels of mobile robots are being developed after analyzing that robots with nonholonomic or holonomic characteristics have a total of three degrees of freedom (two translational degrees of freedom and one rotational degree of freedom).

In this way, mobile mechanisms with holonomic properties can be easily controlled because the wheels can be driven independently and have high omnidirectional mobility. In many mechanisms, an equilateral triangle arrangement structure is used as the basis for roller arrangement [1].

In RoboCup MSL, most robots use mechanisms with three omni rollers. RV-infinity [2], Musashi150 [3], and NuBot [4] are equipped with three omni rollers arranged in an equilateral triangle, achieving omnidirectional movement. In addition, in RoboCup MSL, the kinematics of a ball with two roller velocities as inputs has been proposed as a kinematic study of the ball holding mechanism installed in the mobile mechanism [5]. The kinematics have also been verified using actual machines [6][7]. In terms of robot mobility, research has been conducted to evaluate the efficiency of movement by studying the region of existence of the speed of a mobile robot relative to the roller speed [8][9][10]. In addition, research on the rotation efficiency of a sphere driven by rollers has identified an efficient roller arrangement pattern [11][12].

To verify whether the proposed theoretical formula properly expresses the intended mathematical model, it is necessary to measure it with a measuring device and confirm the error from the theoretical value. However, there are problems with the development cost and time involved in preparing the measuring device. In addition, feedback takes time, so it is thought that it is difficult to shorten the development period.

In previous research, the simulation function of 3DCAD was used to perform simple experiments by performing simulations on a PC from the pre-experimental stage. In a mobile mechanism equipped with three wheels, the trajectory formula was analyzed when the input roller speed was restricted as a constant. Therefore, trajectory analysis when the input roller speed was arbitrary could not be performed [13].

In this study, we derive a trajectory equation when the input roller speed is set to an arbitrary value and use the mechanism analysis function of 3D CAD software to compare it with the derived theoretical equation and verify the consistency of the kinematics.

This study is structured as follows: Chapter 2 analyzes the kinematics and trajectory of a mobile robot equipped with a three-wheel omni roller. Chapter 3 shows the results of verification experiments and simulations. Finally, we provide a summary and future challenges.

2. Analysis of kinematics and robot trajectory

In this chapter, we analyze the robot trajectory for three roller speeds when the roller position is given as a variable. In Section 2.1, we introduce the kinematics for the general triangular arrangement of three rollers. In Section 2.2, we derive the robot trajectory equations assuming an equilateral triangular arrangement.

2.1. *Mobile robot kinematics*

Figure 1 shows a view of the mobile robot that has a common radius of all omni-wheels adapted the *i-th* rollers ($i = 1,2,3$) contact point $\boldsymbol{P_i}$ on a circle, which has a radius R. X_W-Y_W is the global coordinate system (origin $\boldsymbol{O}$) and X_m-Y_m is robot coordinate system (origin $\boldsymbol{\acute{O}}$). And. Roller arrangement angles are α_i .The robot orientation ϕ is referred as the angle between X_W-axis and X_m-axis. and the distance from the robot center to the contact points between wheels and floor is R. Thus, $[v_1, v_2, v_3]^T$ is represented as follows.

Thus, $[v_1, v_2, v_3]^T$ in robot coordinate is represented as Eq. (1).

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -\sin\alpha_1 & \cos\alpha_1 & 1 \\ -\sin\alpha_2 & \cos\alpha_2 & 1 \\ -\sin\alpha_3 & \cos\alpha_3 & 1 \end{pmatrix} \begin{pmatrix} V_{xm} \\ V_{ym} \\ L\dot{\phi} \end{pmatrix} \tag{1}$$

2.2. *Trajectory of mobile robot*

(A) *Calculating the orbital formula*

Consider the roller angular velocity and roller speed at time t [sec]. The relationship between the angular velocity of each roller $\omega_i = \omega_i(t)(i = 1,2,3)$, radius r and $v_i = v_i(t)$ $(i = 1,2,3)$ becomes Eq. (2).

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} r\omega_1 \\ r\omega_2 \\ r\omega_3 \end{pmatrix} \tag{2}$$

We consider equilateral triangular arrangement as $(\alpha_1, \alpha_2, \alpha_3) = (\pi/6, 5\pi/6, 3\pi/2)$. From Eq. (1), the kinematics is can be represented as Eq. (3).

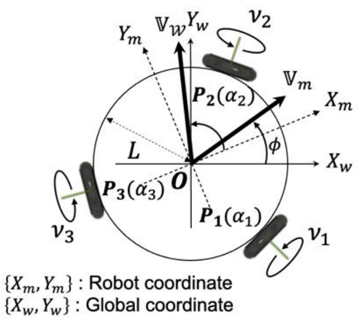


Figure 1 Robot speed vector by robot coordinate system and global coordinate system

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -1/2 & \sqrt{3}/2 & L \\ -1/2 & -\sqrt{3}/2 & L \\ 1 & 0 & L \end{pmatrix} \begin{pmatrix} \dot{X}_m \\ \dot{Y}_m \\ \dot{\phi} \end{pmatrix} \tag{3}$$

The forward kinematics for determining the body speed from the roller input is derived by an inverse matrix as Eq. (4).

$$\begin{pmatrix} \dot{X}_m \\ \dot{Y}_m \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} -1/3 & -1/3 & 2/3 \\ 1/\sqrt{3} & -1/\sqrt{3} & 0 \\ 1/3L & 1/3L & 1/3L \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \tag{4}$$

Also, from Eq. (5),

$$\dot{\phi} = \frac{v_1 + v_2 + v_3}{3L} \tag{5}$$

The transformation from the robot coordinate system Σ_m to the global coordinate system Σ_W can be represented as Eqs. (6)-(7).

$$\dot{\mathbb{P}}_W = \mathbb{T}_{Wm} \dot{\mathbb{P}}_m \tag{6}$$

Where

$$\dot{\mathbb{P}}_W = \begin{pmatrix} \dot{X}_W \\ \dot{Y}_W \\ \dot{\phi} \end{pmatrix}, \ \dot{\mathbb{P}}_m = \begin{pmatrix} \dot{X}_m \\ \dot{Y}_m \\ \dot{\phi} \end{pmatrix}$$
$$\mathbb{T}_{Wm} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

From Eqs. (3)-(6), X_W and Y_W are calculated as Eqs. (8)-(9).

$$X_W = \int_0^t \{ (-v_1 - v_2 + 2v_3) \cos\left(\frac{1}{3L}\int_o^t (v_1 + v_2 + v_3)\,dt\right)$$
$$-(\sqrt{3}v_1 - \sqrt{3}v_2) \sin\left(\frac{1}{3L}\int_o^t (v_1 + v_2 + v_3)\,dt\right)\}\,dt \tag{8}$$

$$Y_W = \int_0^t \{ (-v_1 - v_2 + 2v_3) \sin\left(\frac{1}{3L}\int_o^t (v_1 + v_2 + v_3)\,dt\right)$$
$$+(\sqrt{3}v_1 - \sqrt{3}v_2) \cos\left(\frac{1}{3L}\int_o^t (v_1 + v_2 + v_3)\,dt\right)\}\,dt \tag{9}$$

(B) *Analysis of trajectories in uniform motion*

In this section, we consider the properties of the robot trajectory when the input velocity is constant.

As shown in Figure 2, if the input roller speeds are constant, then we are assuming that$(v_1, v_2, v_3) = (a, b, c)$ $(a, b, c$: constant).

At first. $\|\mathbb{V}_W\|$ is calculated as Eqs. (10)-(12).

$$\|\mathbb{V}_W\| = \sqrt{\dot{X}_W{}^2 + \dot{Y}_W{}^2} = \sqrt{\dot{X}_m{}^2 + \dot{Y}_m{}^2} \tag{10}$$

$$= \frac{1}{3}\sqrt{(-a-b+2c)^2 + \left(\sqrt{3}a - \sqrt{3}b\right)^2} \tag{11}$$

$$= \frac{\sqrt{2}}{3}\sqrt{(a-b)^2 + (a-c)^2 + (b-c)^2} \tag{12}$$

Using $\mathcal{R} = \|\mathbb{V}_W\|/|\dot{\phi}|$, $\|\mathbb{V}_W\|$ is derived as Eq. (13).

$$= \frac{\sqrt{2}}{|a+b+c|}\sqrt{(a-b)^2 + (a-c)^2 + (b-c)^2} \tag{13}$$

Let the velocity vector be $\mathbb{V}_W = (\dot{X}_W, \dot{Y}_W)$, Direction of initial velocity of the robot. It is represented as Eqs. (14)-(16).

$$\psi = \tan^{-1}\frac{\dot{X}_W}{\dot{Y}_W} = \frac{\sqrt{3}a - \sqrt{3}b}{-a-b+2c} \tag{14}$$

Center of circular orbit is represented as Eqs. (15)-(16).

$$\mathbb{O}_R = \dot{\phi}^{-1}\mathbb{C}_{90}\mathbb{V}_W \tag{15}$$

$$= \frac{2L}{a+b+c}\begin{pmatrix}\sqrt{3}a - \sqrt{3}b \\ -a-b+2c\end{pmatrix} \tag{16}$$

$$= \frac{\sqrt{2} * 3L\sqrt{(a-b)^2 + (a-c)^2 + (b-c)^2}}{3(a+b+c)}\begin{pmatrix}\cos(90+\psi) \\ \sin(90+\psi)\end{pmatrix}$$

Therefore. When $a+b+c=0$, $\dot{\phi}=0$, and the robot body moves in a linear motion, When $a+b+c \neq 0$, the motion is circular (Case of $a+b+c>0$, it moves while rotating counterclockwise. Case of $a+b+c<0$, it moves while rotating clockwise).

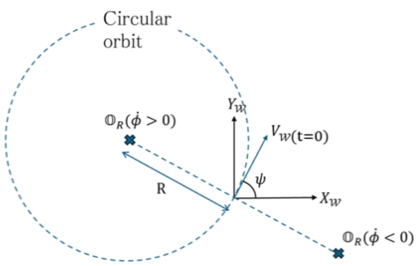


Figure 2 Circular orbit for mobile robot

3. Simulation

3.1. *Set up for simulation*

The simulation was performed using SlidWorks simulation. Figure 3 shows the experimental setup equipped with a three-wheeled omni roller. The assembly diagram includes an isometric view (Figure 3(a)), a front view (Figure 3(b)), and a side view (Figure 3(c)). Table 1 gives the radius of the aircraft body $L = 0.086$[m] and the radius of the omni roller $r = 0.023$[m]. The number of passive wheels of this omni roller is shown, and in this case 18 rollers are assumed (Figure 3(d)). Table 2 shows the roller arrangement conditions. By giving the three roller locations $(\alpha_1, \alpha_2, \alpha_3) = (\pi/6, 5\pi/6, 3\pi/2)$, we assume that the three rollers arranged in an equilateral triangle are oriented in the tangent direction of the robot's periphery. Coulomb friction is set as the contact constraint between the floor and the passive long of the omni roller.

The error rate is expressed using a vector that combines the two-dimensional coordinates of the robot trajectory. The error $e(t)$ is defined as the rate of closeness between the theoretical vector $\mathbb{r}_t(t) = (X_W^m(t), Y_W^m(t))$ and the experimental vector $\mathbb{r}_e(t) = (X_W^e(t), Y_W^e(t))$ integrated over the interval The error rate is expressed using a vector that combines the two-dimensional coordinates of the robot trajectory. The error $e(t)$ is defined as the rate of closeness between the theoretical vector $\mathbb{r}_t(t) = (X_W^m(t), Y_W^m(t))$ and the experimental vector $\mathbb{r}_e(t) = (X_W^e(t), Y_W^e(t))$ integrated over the interval [0, T] at time t.

$$e(t) = \frac{1}{T}\int_0^T \frac{\|\mathbb{r}_t(t) - \mathbb{r}_e(t)\|}{\|\mathbb{r}_t(t)\|} dt \tag{17}$$

Table 1 Set up for mechanism

Parameter	Value
L	0.108[m]
R	0.023[m]
n	18

Table 2 Set up for simulation

Parameter	Name
$\|\mathbb{V}_W\|$	0.022 [m/s]
$(\alpha_1, \alpha_2, \alpha_3)$	$(\pi/6, 5\pi/6, 3\pi/2)$ [rad]

3.2. *Result of simulation*

(A) Simulation 1 (Circular motion)

Consider the motion in which the robot's initial velocity vector is 60° and $\|V\| = 0.022$ [m/s]. Roller input values $(N_1, N_2, N_3, N_3) =$ (32.9, 1.84, 32.9 and (-12.2, -43.3, -12.2) in $T = 3$[s] are values calculated by Eq. (3) for the target direction 60°(Table 3).

As a result, in Figure 4, the theoretical values are shown in orange and the experimental values in blue for comparison.

It is theoretically shown that the trajectory properties lead to a circular motion because $N_1 + N_2 + N_3 \neq 0$ (See [13]).

As shown in Figure 5, the experimental values are close to the theoretical values. The errors are $|1 - L_w^e/L_w^m| = 0.02$ and $|1 - L_w^e/L_w^m| = 0.03$, respectively, demonstrating the accuracy of the kinematics.

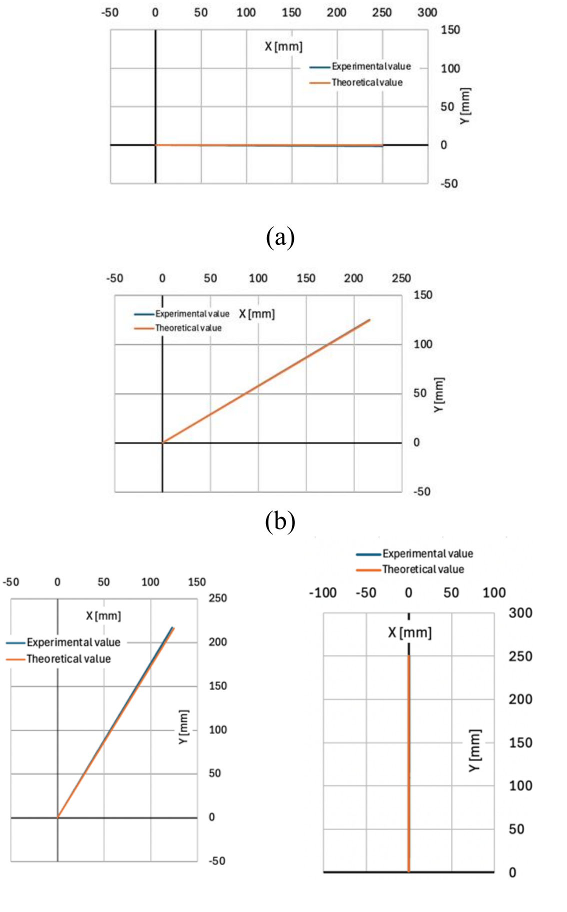
(B) Simulation 2 (linear motion, translational motion only)

Consider motion where the robot speed is $\|V\| = 0.022$ [m/s]. Roller input values are values calculated by Eq. (3) for the target direction 0°, 30°, 60° and 90°. Assuming omnidirectional movement, assume $\varphi = 0°$, $30°, 60°, 90°$ (Table 4).

Although this angle is only in the first quadrant, these four patterns are sufficient due to the symmetry of the roller arrangement of the mechanism. The inputs (N_1, N_2, N_3) are shown in Table 4.

As a result, in Figure 4, the theoretical values are shown in orange and the experimental values in blue for comparison.

Since $N_1 + N_2 + N_3 = 0$, it is theoretically shown that a straight line is drawn due to the nature of the trajectory.



(a)

(b)

(c) (d)

Figure 5 Trajectory of mobile robot in case of constant roller's speed (a) Case of $\varphi = 0°$.(b) Case of $\varphi = 30°$. (c) Case of $\varphi = 60°$.(d) Case of $\varphi = 90$.

As shown in Figure 5, case of $\varphi = 0°$(Figure 5(a)), case of $\varphi = 30°$(Figure 5(b)), case of $\varphi = 60°$(Figure 5(c)), case of $\varphi = 90°$(Figure 5d)), each experimental values are close to the theoretical values. In particular, $Y_w^e(t)$ smoothly approaches $Y_w^m(t)$ (t), and $(X_w^e(t), Y_w^e(t))$ smoothly approaches $(X_w^m(t), Y_w^m(t))$.

The errors are $|1 - L_w^e/L_w^m| = 0.01$, $|1 - L_w^e/L_w^m| = 0.02$, $|1 - L_w^e/L_w^m| = 0.02$, and $|1 - L_w^e/L_w^m| = 0.02$, respectively.

Verification experiments were performed in a simulation environment on a PC, and the accuracy of the kinematics was demonstrated in both circular and linear trajectories.

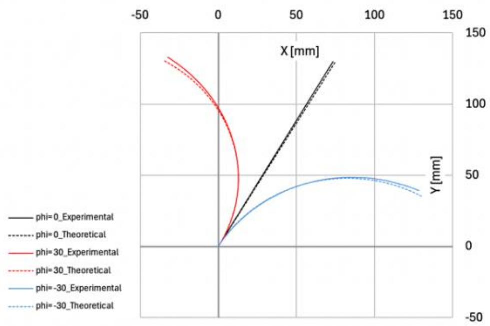
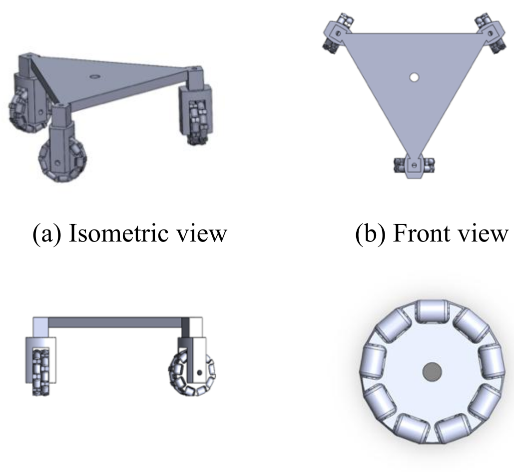


Figure 3 Trajectory of mobile robot in case of constant roller's speed.

Table 3 Input and output roller arrangement

	φ [deg]	(N_1, N_2, N_3)[rpm]	Error
Left rotation	60°	(32.9, 1.84, 32.9)	0.03
Clockwise	60°	(-12.2, -43.3, -12.2)	0.02



(a) Isometric view (b) Front view

(c) Side view (d) Omni roller

Figure 4 Design of mobile robot for simulation

Table 4 Input and output roller arrangement

φ [deg]	(N_1, N_2, N_3)[rpm]	Error
0°	(-10.3, -10.4, 20.8)	0.03
30°	(0, -17.9, 17.9)	0.03
60°	(10.3, -20.8, 10.4)	0.02
90°	(17.9, -17.9, 0)	0.01

4. Conclusion

In this study, the kinematics of a mobile mechanism equipped with three wheels was derived. In order to verify the kinematics, a simulation of the trajectory of the mechanism was carried out using the mechanism analysis function built into 3D CAD software. Cases where the trajectory is circular and where linear motion is assumed in all directions were confirmed, and the error rate was within 0.05, demonstrating the accuracy of the kinematics.

Future works include analyzing the behavior of the mobile robot when the number of omnidirectional wheels or roller passive wheels is increased.

Acknowledgements

This work was supported by JSPS Grant-in-Aid for Scientific Research 23 K13279 and Kyushu Institute of Technology Collaboration fund.

Reference

1. J.Tang, K.Watanabe, et al., Autonomous control for an omnidirectional mobile robot with the orthogonal-wheel assembly, Journal of the Robotics Society of Japan. Vol. 17, No. 1,pp. 51-60, 1999.
2. Y.Yasohara, K.Shimizu et al., Development of ball handling mechanism for RoboCup MSL, 30th Fussy System Symposium, pp. 616-617, 2014.
3. S. Chikushi, M. Kuwada, et al., Development of Next-Generation Soccer Robot"Musashi150"for RoboCup MSL,30th Fussy System Symposium, pp. 624-627,2014.
4. R. Junkai, X. Chenggang, X. Junhao et al., A control system for active ball handling in the RoboCup middle size League, Chinese Control and Decision Conference(CCDC.)2016.
5. K. Kimura, K. Ogata, K. Ishii, Novel Mathematical Modeling and Motion Analysis of a Sphere Considering Slipping, Journal of Robotics, Networking and Artificial Life, Vol.6, issue 1, pp. 27- 32, 2019.
6. K. Kimura, S. Chikushi, K. Ishii, Evaluation of the Roller Arrangements for the Ball-Dribbling Mechanisms adopted by RoboCup Teams, Journal of Robotics, Nrtworking and Artificial Life, Vol.6, issue 3, pp. 183- 190, 2019.
7. S. Chikushi , T.Weerakoon , T.Sonoda , K. Ishii , Kinematics of Two-Roller-Driven Ball for RoboCup Soccer Robot, Journal of Robotics, Networking and Artificial Life, Vol. 4, No. 3, 248–253, 2017.
8. K.Kimura, K.Ishii, The Spherical Robot Transfer Problem With Minimal Total Kinetic Energy, Proceedings of International Conference on Artificial ALife and Robotics (ICAROB2021), Oita 266-270, 2022.
9. K.Kimura,Y.Abematsu, H.Hirai, Kazuo Ishii, Evaluation of Two Rollers Arrangement on a Hemisphere by Kinetic Energy, Journal of Robotics, Networking and Artificial Life, Vol.9, issue 3, pp.233-239, 2022.
10. K. Kimura, K. Ishii, Efficiency Problem of Spherical Robot in Transfer Kinetic Energy,Journal of Robotics, Networking and Artificial Life, Vol.9, issue 1, pp.87-92,2022.
11. K.Kimura, Y.Shigyo, K.Ishii,An Analysis of Robot Speed Efficiency for Mobile Robot Adapted Three Omni Rollers Using Linear Transformation, Journal of Advance in Artificial Life Robotics, Vol.3, issue 4, pp.242-249, 2023.
12. K.Kimura, K. Ishii, Kinetic Analysis of Omnidirectional Mobile Robot with Symmetry Roller's Arrangement, Journal of Robotics, Networking and Artificial Life, Vol.10, issue 3, pp.275-281, 2024.
13. K.Kimura, K.Nakayama, K.Suzuki, K. Ishii, Trajectory Analysis for a Mobile Robot Adapted Three Omni Rollers in Constant Roller's Speed, , International Conference on Artificial Alife and Robotics(ICAROB),pp361-364, 2025.

Authors Introduction

Dr. Kenji Kimura

He is an Associate Professor at the Department of Control Engineering, National Institute of Technology, Matsue College, and a Visiting Associate Professor at Chuo University. He received his MS (Master of Mathematical Science) from Kyushu University in 2002 and his PhD (Engineering) from the Kyushu Institute of Technology in 2020. His research interests are spherical mobile robots.

Mr. Kazuki Nakayama

He is 5th year student at the Department of Control Engineering, National Institute of Technology, Matsue College. His research interests include mobile robot and hand robot.

Dr. Katsuaki Suzuki

He is a Researcher at the Kumamoto Industrial Research Institute, Japan. He received his Ph.D. degree from the Kyushu Institute of Technology in 2021. His research interests include joint mechanisms and their applications.

Dr. Kazuo Ishii

He is a Professor in the Kyushu Institute of Technology, where he has been since 1996. He received his Ph.D. degree in engineering from University of Tokyo, in 1996. His research interests span both ship marine engineering and Intelligent Mechanics. He holds five patents derived from his research.