

Research Article

# Analyzing Learners' Operation Logs in Three-Move Problems with the $2 \times 2 \times 2$ ERC and IDDFS Algorithm

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## ABSTRACT

Japan's high school curriculum now includes "Mathematics through Puzzles" in the "Human and Mathematical Activities" unit to foster spatial and strategic thinking, requiring new teaching tools. While the Rubik's Cube is popular, conventional methods cannot analyze operation records to evaluate learners' thinking. This study uses a  $2 \times 2 \times 2$  ERC and the IDDFS algorithm to analyze operations and introduces an evaluation value to quantify efficiency. We conducted experiments with 11 learners that evaluation value distinguishes the strategic thinking from trial-and-error. This result offers a new alternative objective method to assess cognitive processes in puzzle-based mathematical education in the future.

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## 1. Introduction

In recent years, as the rapid advancement of science and technology and the increasing complexity of social issues have heightened the need for human resource development capable of responding to such changes, educational settings have been drawing attention to STEAM education [1],[2],[3],[4],[5], an interdisciplinary approach that integrates Science, Technology, Engineering, Arts/Liberal Arts, and Mathematics, going beyond traditional cross-curricular learning.

In Japan, the curriculum guidelines for high schools across the country were updated, particularly a new unit "Mathematics and Human Activities" was introduced in the high school subject "Mathematics A." This unit aims to develop students' ability to apply knowledge and creative problem-solving skills [6]. Within this unit, "Mathematics in Games and Puzzles" features examples such as magic squares, tic-tac-toe, and the Rubik's Cube, encouraging activities that promote mathematical thinking through hands-on experience.

Against this backdrop, we developed the  $2^3 - ERC$ , a Rubik's Cube-type educational tool featuring  $2 \times 2$  blocks on each face, to assess learners' mathematical thinking through manipulative experience [7]. However, evaluating a Rubik's Cube is difficult due to the existence of multiple solution paths and the complexity of analyzing them. Therefore, the IDDFS (iterative deepening depth-first search, IDDFS) algorithm is used to find the shortest path, and its evaluation value is used as an indicator of understanding.

This study analyzes three-move problem manipulation data collected at the National Institute of Technology, Matsue College, using IDDFS algorithm. We evaluate the value to distinguish strategic thinking from trial-and-error thinking.

## 2. $2^3 - ERC$ Teaching System

### 2.1. Overview of the $2^3 - ERC$

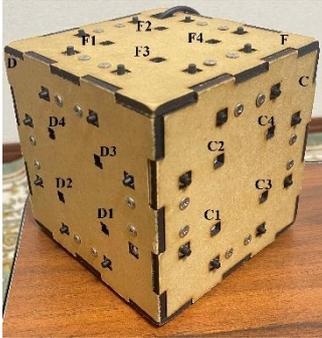
The  $2^3 - ERC$  is an Electronic Rubik's Cube-type educational tool that can be connected to a computer to collect experimental data [7]. The device is cube-shaped, and each face is equipped with four LEDs and eight control

buttons. Fig.1 shows the layout of the  $2^3 - ERC$  surface and LEDs.

When a learner presses the control buttons, the LED color patterns change in a manner analogous to the rotational movements of a Rubik's Cube. By having learners manipulate this tool, instructors aim to cultivate skills such as spatial and strategic thinking patterns.



a. A-B-E side view



b. C-D-F side view

Fig.1 Physical configuration of the LED-to-face assignment in the  $2^3 - ERC$

## 2.2. Mathematical Models

Fig.1 shows each face of the  $2^3 - ERC$  is assigned a face identification index from surface - A to surface - F. Using these indices, identifiers are assigned to the LEDs; Fig.1(a) show the A - B - E side view, for instance on face A, the upper-left LED is defined as  $A_1$ , and the lower-right LED as  $A_4$ . Then, it goes the same with Fig.1(b) when C - D - F side view. To handle the color information of these LEDs, a vector  $q_n$  with 24 elements is defined as shown in Eq. (1). The color information of the LEDs represented by  $q_n$  is treated as color indices corresponding to each color.

$$q_n = (A_1, A_2, A_3, A_4, \dots, F_1, F_2, F_3, F_4) \in \mathbb{R}^{24} \quad (1)$$

Next, the rotation rule for rotating  $q_n$  is given by Eq. (2), where Eq. (2) represents a permutation matrix.

$$T_X, T_Y, T_Z, T_{X^{-1}}, T_{Y^{-1}}, T_{Z^{-1}} \in \mathbb{R}^{24 \times 24} \quad (2)$$

The fundamental rotation rules of the  $2^3 - ERC$  consist of six types: three forward rotations  $T_X, T_Y, T_Z$ , and their

corresponding inverse rotation  $T_{X^{-1}}, T_{Y^{-1}}, T_{Z^{-1}}$ . It should be noted that the identity transformation  $E$  is not included among the fundamental rotation rules. Using these six types of rotations, the motion of the  $2^3 - ERC$  can be represented. Therefore, the transformation matrix  $T$  corresponding to an arbitrary rotation can be defined as shown in Eq. (3).  $T$  is the rotation matrix and  $q_0$  is the initial state. Then,  $q_0$  is transformed into the  $n$ -th state  $q_n$  by  $T$ .

$$q_n = Tq_0 \in \mathbb{R}^{24}$$

$$T \in \prod_{i \in S} X_i (S \subseteq \{T_X, T_Y, T_Z, T_{X^{-1}}, T_{Y^{-1}}, T_{Z^{-1}}\}) \quad (3)$$

Where  $|S| > 0$ .

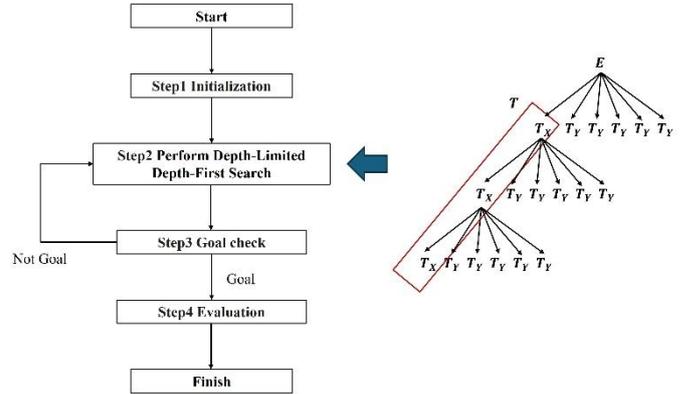


Fig.2 Flowchart of the System and Solution Search by IDDFS when  $T = T_X T_X T_X$  is the Goal State (redrawn based on [7])

## 3. Solution Search and Evaluation Using IDDFS algorithm

The operation data collected from learners using the  $2^3 - ERC$  must be evaluated using an appropriate analytical method. Although various approaches can be considered for evaluating the data obtained from the  $2^3 - ERC$ , this study adopts the IDDFS algorithm and evaluates the data based on the shortest path obtained through this approach.

Fig.2 shows the flowchart of the proposed system and the solution search mechanism using IDDFS algorithm when  $T = T_X T_X T_X$  is the solution.

The IDDFS-based solution search and evaluation process are described in Steps 1 - 4 below.

### Step 1. Initialization

The solution  $\hat{T}$  to be obtained by IDDFS algorithm is initialized as the identity matrix, and the search depth limit is set to 0.

### Step 2. Depth-limited depth-first search

Depth-first search is performed up to the restricted depth limit. When the restricted depth is reached, the search is terminated.

### Step 3. Goal state check

If  $T\hat{T}$  matches the target state, the search is completed and proceeds to Step 4. Otherwise, the depth limit is incremented by one, and the process returns to Step 2.

### Step 4. Evaluation

The evaluation function  $L$  is defined as the difference between the sum of the ideal minimum number of rotations for each state operated by the learner and the sum of rotations required to reach the ideal state from the initial state, as shown in Eq. (4).

$$L = -\frac{\alpha(\alpha + 1)}{2} + \sum_{i=1}^m \beta_i [-], \quad i \in \mathbb{Z}_+ \quad (4)$$

Here,  $L$  is dimensionless,  $\beta_i$  represents the minimum number of rotations required to reach the target state from the current permutation state  $\hat{T}$  using unit rotations,  $\alpha$  denotes the number of moves required to reach the target state from the initial state given as the task, and  $m$  indicates the number of operations performed by the learner up to the current point.

## 4. Methodology and Experimental Procedure

To verify the effectiveness of the proposed method, an experiment was conducted with 11 students from National Institute of Technology, Matsue College. The students who participated in the experiment had previously responded that they did not know how to solve a Rubik's Cube. In this experiment, the initial state of the  $2^3 - ERC$  was set as  $T_x T_y T_z$ , and participants were evaluated based on the time and operation sequence required to return the device to the target state  $E$ . The experiment continued until each student either completed the given task or chose to discontinue participation by voluntarily requesting to terminate the experiment.

## 5. Results and Discussion

In this study, a task-based experiment using the  $2^3 - ERC$  was conducted with 11 students from National Institute of Technology, Matsue College. The number of operations and time required to reach the target state from the initial state were recorded and compared with the shortest solution obtained using the IDDFS algorithm. Table.1 shows experiment results. The experimental results showed that the students could be broadly divided into two groups, one group completed the task with fewer operations and in a shorter time, while the other required more operations and longer time. The former group is likely to have understood the rotation rules of the Rubik's Cube, constructed logical and efficient operation sequences, and successfully derived the solution. In

contrast, the latter group tended to engage in trial-and-error behavior, indicating that strategic thinking necessary to reach the goal state was not sufficiently demonstrated.

Furthermore, analysis using the evaluation function  $L$  revealed fluctuations in cognitive processes that conventional metrics such as operation count and completion time could not capture. For instance, participants whose operation sequences frequently deviated from the target state achieved lower scores, reflecting suboptimal manipulation quality. Conversely, participants who executed operations efficiently achieved high scores, suggesting they may possess strong problem-solving and pattern recognition abilities. These results demonstrate that analyzing the  $2^3 - ERC$  operation log provides an objective framework for understanding not only task performance but also the underlying cognitive mechanisms of learners.

Table.1 Total Experiment Time (sec), Number of Operations, and Evaluation Function  $L$  for 11 Students

	Total time [sec]	Number of manipulative moves	Evaluation function $L$
S1	8	3	6
S2	299	92	890
S3	294	44	375
S4	296	113	975
S5	13	3	6
S6	299	63	616
S7	17	5	9
S8	26	5	11
S9	299	37	281
S10	12	6	6
S11	10	6	6

## 6. Conclusion and Future Work

In this study, we developed a  $2^3 - ERC$  and proposed a method for analyzing its operation logs using the IDDFS algorithm.

Through an experiment involving 11 students from National Institute of Technology, Matsue College, we demonstrated that the proposed method could evaluate learners' operation tendencies.

The main findings are summarized as follows

- By comparing the operation logs of the  $2^3 - ERC$  with the optimal solution, the efficiency of operations could be quantitatively evaluated.
- By introducing the evaluation function  $L$ , it became possible to assess not only the number of operations and time required, but also the nature of understanding based on the learner's sequence of thought.

In the future, we aim to increase the number of participants and conduct verification in various

educational settings, as well as improve both the hardware and software of  $2^3 - ERC$  and introduce methods to reduce the possibility of incorrect judgments based on accidental solutions, with the aim of developing it into a more versatile and practical mathematics education support tool.

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