Research Article

Mathematical Modeling and Simulation of a Simple Pendulum-Based Hide Tanning Equipment

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ABSTRACT

Limited access to electricity in rural areas of developing countries has driven innovation in gravity-assisted human-powered mechanical systems. This study presents the mathematical modeling and simulation of a newly designed hide tanning device that utilizes a simple pendulum mechanism powered by human muscle force. The equipment integrates a simple pendulum, cylindrical gear transmission, bearings, and hide straps into a dynamic system. Equations of motion were derived using the Lagrangian method, validated through numerical simulation over a ten-second period. The torque resistance of the hide strap was evaluated using finite element analysis (FEA) in a previous study, enabling the calculation of the strap's resistance coefficient. Simulation results demonstrated the damping effect caused by the hide straps on the pendulum's oscillation and quantified the required excitation force applied by hand. The device is capable of tanning up to six hide straps simultaneously. The findings confirm that the proposed equipment effectively reduces physical effort, improves labor efficiency, and offers a sustainable and user-friendly solution suitable for traditional hide processing in remote and off-grid communities.

1. Introduction

Access to reliable electricity remains a major challenge in rural areas of many developing countries. According to a World Bank report released on October 18, 2024, approximately 700 million people worldwide live without electricity [1]. The 2024 extended report on the Sustainable Development Goals, published by the United Nations Department of Economic and Social Affairs, states that as of 2022, 85% of those without electricity lived in rural areas [2]. Additionally, a satellite survey using 3,000 nighttime images estimated that at least 1.18 billion people have experienced energy shortages globally [3]. In Mongolia, people living in rural regions, especially nomadic livestock herders still lack full access to electricity. Even today, herders continue to produce everyday goods using physically demanding traditional methods powered solely by human muscle. To reduce this labor intensity, there is a growing need for small-scale, low-tech equipment that utilizes renewable or gravity-based energy, conserves physical effort, and improves the quality of life for herders. Herders, who preserve Mongolia's nomadic culture, depend on five types of livestock and migrate seasonally in search of pasture. According to the National Statistics Office of Mongolia,

the total livestock population increased from 43,288,500 in 2008 to 57,647,900 in 2024 an increase of 24.9%. In contrast, the number of active herders declined from 359,597 to 308,730 over the same period, representing a 14.14% decrease [4]. Although the livestock population has grown, the number of individuals engaged in hide processing and other traditional activities has declined, putting at risk many heritage-based practices. Traditional tanning methods primarily include hand tanning, wringing, spinning, and fanning as shown in (Figure 1). These techniques are labor-intensive and rely entirely on human muscle force.
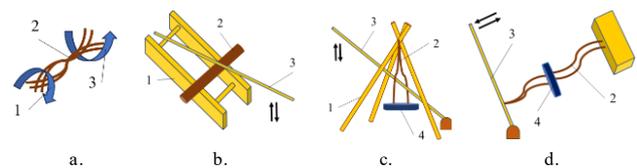

Figure 1 a-hand tanning, b-wring, c-spinning, d-fanning

• Hand tanning: A short-sized hide strap (2) is twisted and turned using both hands (1, 3). This method requires considerable hand strength and causes friction on the

palms, making the process physically demanding as illustrated in (Figure 1a).

• Wring tanning: The hide (2) is bundled with other straps and held horizontally (1), while a wooden stick (3) is used to repeatedly press down. The straps are turned after each press. This requires a significant amount of force from both hands as shown in (Figure 1b).

• Spinning tanning: The hide strap is attached to the upper end of a three-point structure (1), with a weighted load (4) suspended at the bottom to induce twisting. A wooden stick (3) moves up and down to enhance torque and soften the hide. This technique requires both hand and back strength as shown in (Figure 1c).

• Fanning tanning: The hide strap (2) is looped horizontally, with one end fixed and a twisting fan (4) in the center. Pulling the strap from one end requires full-body effort, making it an arduous task as illustrated in (Figure 1d).

Despite their cultural importance, these traditional methods are strenuous and inefficient, particularly for aging herders or those with limited physical capacity. To address these challenges, this study explores the use of a gravity-based mechanism, a simple pendulum as the core of a novel hide tanning system. Pendulums offer smooth, repetitive oscillatory motion that can convert intermittent human input into consistent mechanical output. Historically, pendulum mechanics have been applied in several innovations. Zhang Heng, a Chinese Han Dynasty scientist, developed a pendulum-based seismometer in the 1st century to detect earthquakes [5]. Galileo Galilei began his foundational studies of pendulum motion in 1602 [6], and Christiaan Huygens invented the first pendulum clock in 1656 [7, 8]. In recent years, pendulum-based systems have been adapted for water pumping in energy-limited environments by utilizing human hand force to initiate and maintain oscillation [10–13]. Yakubu, Olejnik, and Awrejcewicz demonstrated a water-pumping device powered by a variable-length pendulum [14]. Further studies have explored integrating pendulums with other sources of motion such as ocean waves, human locomotion, and ambient vibration to accumulate and stabilize energy [15]. According to Mongolian tradition, hides are considered symbols of wealth and prosperity. Herders have long produced hide products such as bridles, halters, and hobbles from cattle, horse, camel, and goat skins due to their durability. However, the manual labor required by conventional tanning methods threatens the sustainability of this heritage, especially as modernization reduces the rural workforce. Even today, herders rely solely on human muscle force for hide processing. This study presents the mathematical modeling and simulation of a newly designed hide tanning device that integrates a simple pendulum with human input. This approach bridges traditional craftsmanship with modern mechanical design, offering a sustainable solution for off-grid communities.

2. Methodology

2.1. System architecture and kinematic model

As of 2024, Mongolia is home to 249,456 herder households living nomadically across its vast territory of 1,564,100 square kilometers [4]. The leather-tanning device proposed in this study has potential applications across these households, as well as in small enterprises engaged in leather goods production and by individuals interested in traditional leathercraft. A key advantage of the device is its ability to significantly reduce the physical effort required during the tanning process, owing to its mechanically simple yet effective design. Its user-friendly operation allows individuals of varying physical capacity including children, and the elderly to operate safely and efficiently. According to a comprehensive review of existing literature, no previous study has introduced or developed a leather-tanning device based on a simple pendulum mechanism.
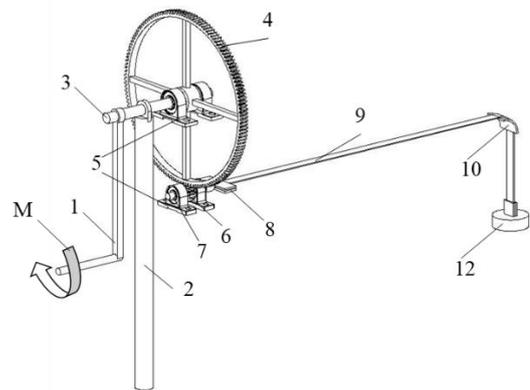

Figure 2 Three-dimensional model of the proposed hide-tanning device incorporating a simple pendulum mechanism

The proposed device comprises three primary subsystems: a simple pendulum (2), a cylindrical gear transmission (4,6), and the leather strap or hide (9). In addition to these, the system includes several supporting components: a manual handle (1), an output shaft (3), two upper bearings (5), a driven shaft (7), two lower bearings (5), a leather-fixing head (8), a sliding support mechanism (10), and a counterweight (12). A three-dimensional physical model illustrating the configuration and arrangement of these components is presented in (Figure 2). The equipment was ergonomically designed based on the average height and reach of Mongolian adults to ensure usability across age groups, and it operates entirely by manual force, making it suitable for off-grid, rural environments where electricity is limited.

Table 1 Geometric and physical properties of the simple pendulum

Parameter	Symbol	Value	Unit	Source/Note
Pendulum length	1	0.8	M	Based on average adult height [4]
Pendulum diameter	–	0.05	M	Design specification
Pendulum density (steel)	ρ	7800	kg/m³	Assumed for solid cylinder
Calculated pendulum mass	m	12.252	Kg	From volume and density
Initial angle of oscillation	θ_0	120°/2.094	degrees/rad	Initial condition for simulation
Gravitational acceleration	g	9.81	m/s²	Constant

The physical and dynamic parameters used in the mathematical modeling of the hide-tanning device are summarized in (Table 1 and 2).

Table 2 System resistance and mechanical properties of the leather strap

Parameter	Symbol	Value	Unit	Source/Note
Bearing friction coefficient	$C_1=C_2=C_3=C_4$	0.02	-	(5) and (7) in (Figure 2)
Hide strap resistance coefficient	R_L	0.33	-	From FEA [17]
Hide strap density	ρ	780	kg/m³	From FEA [17]
Young's modulus	E	100.24	MPa	For traditionally processed cow leather strips
Poisson's ratio		0.35		-
Hide strap segment with dimensions	[h,w,l]	3x30x124.45	Mm	h – thickness, w – width, l – length

These include the pendulum dimensions, material properties, and initial simulation conditions, as well as system-specific coefficients such as bearing friction and strap resistance. The excitation force values, determined based on varying numbers of hide straps, reflect the user input required to maintain oscillation. These values were either derived analytically, assumed based on mechanical standards, or validated through finite element analysis (FEA).

2.2. System dynamics

The motion of the simple pendulum was modeled using Lagrange method that led to the nonlinear differential equation:

$$\ddot{\theta} + \frac{2g}{l}\sin\theta = 0 \qquad (1)$$

The inertia of the gear transmission was included by modifying the moment of inertia. The equation becomes:

$$\ddot{\theta}\left(\frac{1}{2}M_1R_1^2 + m\left(\frac{l}{2}\right)^2 + \frac{1}{2}M_2R_1R_2\right)$$
$$+ mg\frac{l}{2}\sin\theta = 0 \qquad (2)$$

The equation of motion for the simple pendulum, when coupled with a cylindrical gear transmission, was derived using Lagrangian methods. The consistency of the results was confirmed by simulation. The governing equation is expressed as:

$$C_1\dot{\theta} + C_2\dot{\theta} + \ddot{\theta}\left(\frac{1}{2}M_1R_1^2 + m\left(\frac{l}{2}\right)^2 + \frac{1}{2}M_2R_1R_2\right)$$
$$+ mg\frac{l}{2}\sin\theta + C_3\dot{\theta}\frac{R_1}{R_2} + C_4\dot{\theta}\frac{R_1}{R_2} + R_L\dot{\theta}\frac{R_1}{R_2}$$
$$= 0 \qquad (3)$$

Where θ – angular displacement (rad), $\dot{\theta}$ -angular velocity of the driving gear and pendulum (rad/s), $\ddot{\theta}$ – angular acceleration (rad/s²), m – mass of the pendulum, M_1 – mass of the driving gear (5.844 kg), R_1 – radius of the driving gear (0.3368 m), M_2 – mass of the driven gear (0.208 kg), R_2 – radius of the driven gear (0.02375 m). Bearings are installed at both the output and driven shafts to reduce friction and support axial loads. The twist resistance coefficient of the hide strap is determined as the ratio of the internal torque of the hide strap to the external torque applied by the hide strap tanning equipment.

$$\varsigma_{cэ} = \frac{M_{lsi}}{M_{lso}} \qquad (4)$$

Here: M_{lsi}- is internal torque of the hide strap, M_{lso}- external torque of the hide strap. Then using Equation 5,

the external torque of the hide strap was determined, using the gravitational moment of a simple pendulum as

$$M_{lso} = -\frac{l}{2}mgsin\theta \qquad (5)$$

To determine the excitation force applied by hand the hide strap tanning equipment is designed to oscillate within ±120° range to effectively tan the hide strap and need to determine the excitation torque to compensate for hide strap twisting and bearing friction damping. The excitation torque is determined by subtracting the internal torque M_{FM} with bearing friction and hide twisting from the torque M_P of a simple pendulum as expressed:

$$M_{RF} = M_P - M_{FM} \qquad (6)$$

Where M_P is the torque of the pendulum (Nm). M_{FM} - is the internal torque with bearing friction and hide strap twisting resistance (Nm). To calculate the hand force needed to sustain motion, we use the net torque method

$$M_P = -\left(\frac{l}{2}mgsin90^0 + \frac{l}{2}mgsin30^0\right) \qquad (7)$$

3. Results

3.1. Pendulum behavior under dynamic conditions

(Figure 3a) illustrates the force balance on the pendulum bob: gravitational force (mg), tension (T), and the tangential restoring component (mg·sinθ). These forces form the basis for the Lagrangian derivation in Eq. (1).
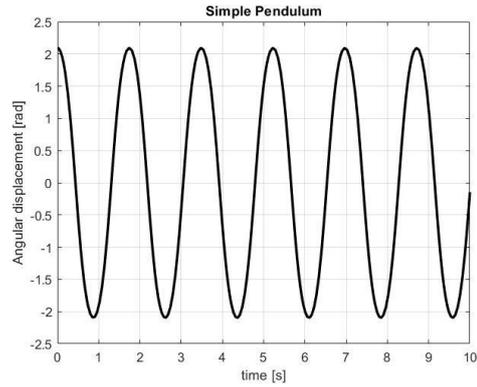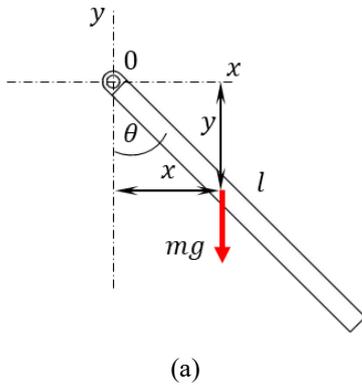
(a)

(b)

Figure 3 (a) The diagram of forces in equilibrium for a simple pendulum; (b) Oscillation of a Simple Pendulum

For a simple pendulum with cylindrical gear transmission in (Figure 3a), the oscillation occurs with an amplitude of 2.094 radians, and the first cycle lasts 1.86 seconds, completing 5.25 cycles in 10 seconds as shown in (Figure 3b).
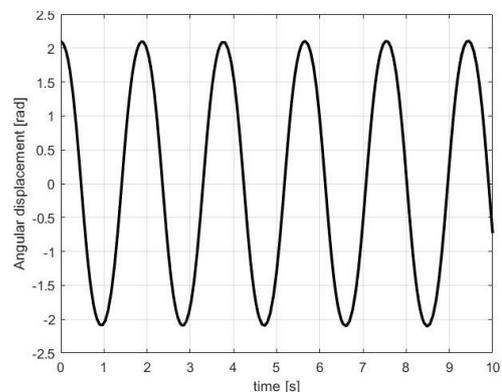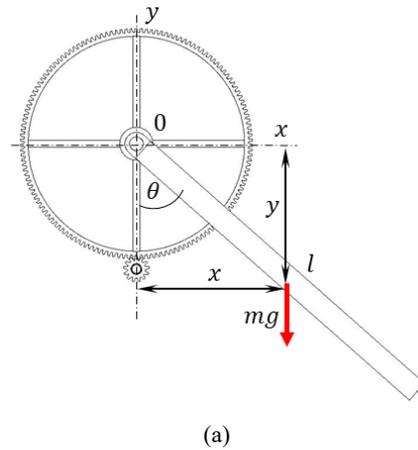
(a)

(b)

Figure 4 (a) Force Equilibrium Diagram of a Simple Pendulum with Cylindrical Gear Transmission; (b) Oscillation of a Simple Pendulum with Cylindrical Gear Transmission

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For a simple pendulum with cylindrical gear transmission in (Figure 4a), the oscillation occurs with an amplitude of 1.562 radians, and the first cycle lasts 1.98 seconds, completing 4.85 cycles in 10 seconds as shown in (Figure 4b). The cylindrical gear transmission modifies the pendulum's kinematics by increasing the rotational speed of the driven shaft while reducing torque. Compared to the simple pendulum in (Figure 3), the gear transmission increases the first-cycle period by 0.12 seconds (6.45%) and reduces the amplitude by 0.532 radians (25.38%). This configuration enables multi-strap processing while maintaining stable oscillatory behavior.

(Figure 5a) integrates all resistive elements into the dynamic model. This model enables accurate prediction of damping behavior. The diagram visualizes the opposition between the driving gravitational torque (M_p) and the resistive internal torque (M_{FM}), which is critical for calculating the net excitation force required from the user as defined in Equation (6).
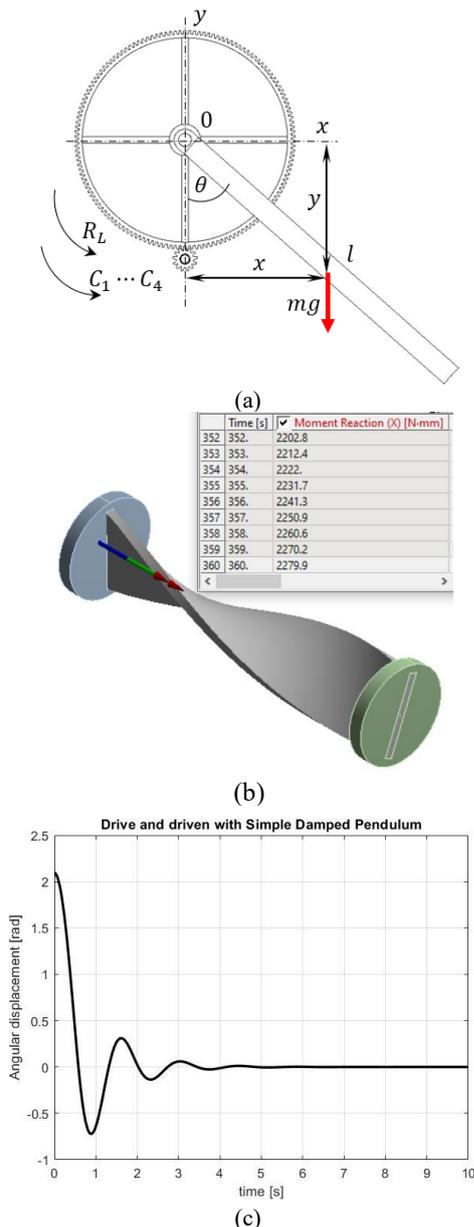

(a)

(b)

(c)

Figure 5 (a) Force equilibrium diagram of a simple pendulum with cylindrical gear transmission; (b) Internal torque of the hide strap; (c) Oscillation of a simple pendulum with cylindrical gear transmission, bearing friction and hide strap resistance was considered

During half of the oscillation period of a simple pendulum, the hide strap twists eight times along its length axis and make hide softening. Since the hide strap undergoes a sequential helical twist along its length in eight times, the internal torque of the hide strap can be expressed in terms of a single twist. The length corresponding to a full 3600-degree rotation of the hide strap was measured to be 124.45 mm. Given parameters in (Table 2) that the hide strap has a density of 780 kg/m³, a Young's modulus of 100.24 MPa, and a Poisson's ratio of 0.35 experimental result was obtained and intro-duced in [17], the internal torque of a hide strap segment with dimensions 3x30x124.45 mm was determined using FEA to be 2279.9 N·mm in (Figure 5b). For the gravity-driven hide tanning equipment, it is designed to process a maximum of six straps of hide at most. The internal torque of a single hide strap has been determined to be 2.2799 N·m. When multiplied by six straps, the total internal torque generated by all the hide straps is 13.6794 N·m. By comparing the internal torque of the hide strap to the external torque, the resistance coefficient of the hide strap has been determined to be 0.33. This result is derived from FEA (Finite Element Analysis) simulations; however, in reality, factors such as the hide strap's moisture content, thickness, cuts, the age and sex of the livestock, and their physical condition can cause differences. When the initial angular condition of the simple pendulum with cylindrical gear transmission is θ(120) (or 2.094 radians), then the oscillation for 10-second simulation period is shown in (Figure 5c).
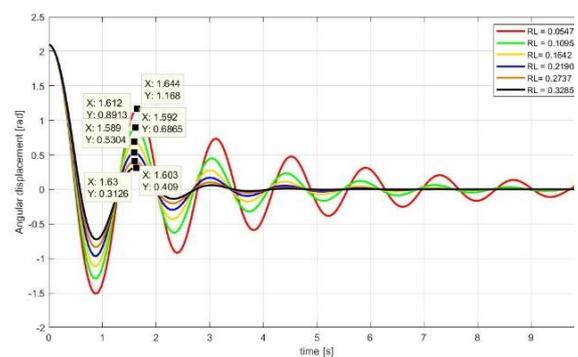

Figure 6 Damped motion of a simple pendulum belongs to each resistance coefficient of the hide strap

According to the damped motion graph in (Figure 6), it is observed that as the number of hide straps increases, the amplitude of the pendulum's oscillation decreases. Result shows that the force diagram of the pendulum system when incorporating the effects of gear transmission friction, bearing friction, and leather strap resistance. These combined factors introduce damping into the system,

resulting in a gradual reduction in oscillation amplitude over time. Using the simulation parameters listed in (Table 1), and applying Equation (5), the external torque acting on the hide strap was calculated based on the gravitational moment of the pendulum. The computed external torque was 41.64 Nm. This value was then used to determine the internal torque comprising both bearing friction and strap resistance for each strap configuration. The resulting internal torque and corresponding excitation moment (i.e., the manual torque required from the user) are summarized in (Table 3).

Table 3 Excitation force and internal torque based on the number of hide straps

Number of Hide Straps	Maximum Amplitude (rad)	Maximum Angle θ_{FM} (°)	Excitation force, F_{RF} (N)
1	1.168	66.92	70.9
2	0.8913	51.07	87.72
3	0.6865	39.33	104.55
4	0.5304	30.38	119.57
5	0.409	23.43	133.4
6	0.3126	17.91	143.37

3.2. *Required excitation force applied by hand*

The excitation force required to maintain the oscillation of the pendulum under different hide strap loads was analyzed using both analytical calculations and finite element analysis (FEA). As shown in (Table 3), the angular amplitude of the pendulum decreases with an increasing number of hide straps, resulting in a corresponding increase in the internal torque and required excitation force.

For a single hide strap, the excitation force was calculated to be 70.9 N (7.13 kg), while for six straps, the required force increased to 143.37 N (14.61 kg). This trend demonstrates a near-linear relationship between the number of straps and the applied manual force. The internal torque caused by hide resistance and bearing friction was determined from the maximum angular displacement of the first oscillation cycle, extracted from the damped motion simulation in (Figure 5c). The torque differential between the external gravitational moment of the pendulum and the internal system resistance was used to compute the excitation torque, which was then converted into the required manual force. This force falls within the physical capacity of most adults, suggesting that the device is ergonomically feasible for manual operation even in rural, off-grid conditions.

4. Conclusion

This study proposes and analyzes a newly designed hide tanning device that combines human hand force with a simple pendulum mechanism to reduce muscular effort in traditional hide processing. The dynamic behavior of the system was modeled using Lagrangian methods, and the oscillatory motion of the pendulum was evaluated under various mechanical conditions.

The required manual force for operation was found to range between 7.13 kg and 14.61 kg, depending on the number of hide straps processed. The resistance coefficient of the hide twisting was determined using finite element analysis (FEA), with the understanding that this value may vary due to factors such as livestock age, sex, body fat, geographic origin, tanning condition, and moisture content. The resulting equipment features a simple and compact structure, requires no external power source, and is easy to operate and maintain. It is designed for off-grid environments and can be used by individuals of all ages, both indoors and outdoors. The proposed system offers a sustainable, ergonomic solution for traditional hide tanning in rural and nomadic communities.

Future work will focus on experimentally validating the hide resistance coefficient, fabricating and testing the physical prototype, comparing manual input requirements with those of traditional hand-tanning techniques, and exploring the integration of renewable energy systems and intelligent control using machine learning.

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