

Research Article

Interpretive Gaps and Minimal Supplementation in Dialogue: A Category-Theoretic Perspective

Yukiko Furuya^{1*}, Akinori Abe²

¹Graduate School of Informatics, Chiba University, 1-33 Yayoicho, Inage Ward, Chiba, 263-8522, Japan

²Faculty of Letters, Chiba University, 1-33 Yayoicho, Inage Ward, Chiba, 263-8522, Japan

Email: 25RD1001@student.gs.chiba-u.jp

*Corresponding Author

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ABSTRACT

Dialogue is susceptible to interpretive divergence. This paper introduces a category-theoretic framework for analyzing interpretive gaps in dialogue and characterizing the minimality of their repair. Building on our empirical study of improvisational acting, we distinguish two types of divergence: passive gaps, arising from cumulative drift of object representations across a semiotic chain, and active gaps, resulting from intentional abductive reconstruction of the interpretive frame. Within a unified categorical formalism, these correspond to object drift under a fixed analogical functor and functor switching, respectively. We characterize minimal supplementation via the universal property—a supplement is minimal if every other valid supplement factors uniquely through it—and prove its uniqueness up to isomorphism. Empirical grounding is provided through 106 inference episodes from five dyads, revealing significant associations between inference type and gap type ($\chi^2(6) = 32.59$, $p < .01$, Cramér's $V = 0.39$).

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1. Introduction

1.1. Background

Interpretive discrepancies in dialogue are unavoidable. While Clark [1], Schegloff [2], and Ginzburg [3] address grounding, repair structure, and formal dialogue semantics, and recent interactional accounts have framed repair as a continuous coordination mechanism rather than a localized correction [4, 5], the "minimality" principle governing supplement selection still lacks formal characterization. Building on Abe's abductive model [6], and grounded in foundational category theory [7, 8], this study formalizes minimal supplementation via category-theoretic universality.

Despite four decades of empirical research on conversational grounding and repair, a principled formal account of the minimality principle governing supplement selection has remained elusive. Grice's Maxim of Quantity [9]—"make your contribution no more informative than required"—implicitly invokes minimality, yet offers no formal mechanism for adjudicating among competing supplements. Similarly, while Relevance Theory [10] provides a processing-effort perspective, it does not characterize the structural relationships among competing hypotheses in a way that yields uniqueness guarantees.

Existing formal approaches each capture partial aspects of the phenomenon. Set-theoretic formulations equate "minimal" with smallest cardinality, thereby ignoring the logical entailment structure among candidates. Information-theoretic formulations minimize a measure such as entropy or description length, but the result depends on the choice of measure and cannot express composability of sequential supplements. Logical abduction models, including the framework of Abe [6], rigorously characterize hypothesis formation but stop short of providing a universal criterion that singles out one supplement as definitively minimal. Category theory [7, 8] addresses all three limitations simultaneously, and has been applied to semantic compositionality [11], which motivates its use in the present work.

1.2. Prior Work: Analysis of Interpretive Gaps

Our prior work [12] analyzed interpretive gaps via Peirce's semiotics in an improvisational acting experiment, identifying two types: (1) passive gaps—unintentional drift of object representation through cumulative semiosis, and (2) active gaps—intentional generative of new interpretants via abduction. A significant association between inference types and gap types was observed ($\chi^2(2) = 6.73$, $p \approx .03$), but the minimality principle in supplement selection remained uncharacterized.

- **Passive gap:** A phenomenon in which the representation of the object drifts as an accompanying effect of the stepwise updating of interpretants during the cumulative progression of semiosis. This is an unintentional interpretive divergence between interlocutors.
- **Active gap:** A phenomenon in which a new interpretant is intentionally generated, steering a branching of the semiotic chain through creative hypothesis formation (abduction). This is an active reconstruction of interpretation by the interlocutor.

In that study, a significant association was observed between inference types (abduction, deduction, induction) and the type of gap ($\chi^2(2) = 6.73, p \approx .03$). However, the minimality principle—i.e., why certain supplements are selected—was not formally characterized.

1.3. Objective of This Study

This study unifies the two gap types and formally characterizes minimality in dialogue repair using category theory. As reviewed in Section 1.1, existing approaches—set-theoretic formulations, information-theoretic minimization, and logical abduction models (including Abe [6])—each capture only partial aspects and share a common weakness: none can guarantee uniqueness of the selected supplement. Category theory [7, 8] overcomes all three limitations simultaneously by offering: (1) minimality via universality—a supplement is minimal if every other valid supplement is reachable from it via a morphism, with provable uniqueness (Theorem 1); (2) compositionality—morphism composition naturally represents sequential multi-turn repairs; (3) functor-based analogy—enabling a unified formal treatment of both passive and active gaps within a single category-theoretic framework [3], [11].

1.4. Organization of This Paper

Section 2 presents the theoretical framework, and Section 3 connects it to prior work. Section 4 reports the results of preliminary validation, and Section 5 discusses implications and future directions.

Specifically, the contributions of this paper are as follows. First, we introduce a category-theoretic formalization of dialogue states, interpretive gaps, and supplementation, building on the dialogue-state category whose objects are knowledge states (clause sets) and whose morphisms represent inferential entailment. Second, we define minimal supplementation via the universal property—a notion that does not depend on any particular information measure and yields provable uniqueness (up to isomorphism). Third, we show that the two empirically identified gap types (passive and active) map cleanly onto structural features of the categorical formalism: object drift and functor switching, respectively. Fourth, we provide a preliminary empirical validation demonstrating that the formalism's predictions are consistent with observed

inference patterns, and we offer three detailed case analyses illustrating the formalism in action.

2. Theoretical Framework

2.1. Repair as Abduction

Following Peirce's classical account of abductive inference [13], a hypothesis is introduced whenever a surprising observation cannot be accounted for by what is already known. Abe [6] provided a precise logical formulation of this reasoning pattern, which we adopt here:

- **Observation:** C (the fact to be explained)
- **Background knowledge:** Σ (a known knowledge base)
- **Condition:** $\Sigma \not\vdash C$ (C is not derivable from Σ)
- **Hypothesis:** find H such that $\Sigma \cup H \vdash C$

Projected onto the conversational setting: C denotes the partner's utterance or action whose significance requires explanation; Σ represents the common ground accumulated up to that point in the exchange; and H is the tacit assumption that, once introduced, renders the partner's contribution coherent within the shared discourse.

2.2. Category-Theoretic Formalization

Definition 1 (Dialogue-State Category). **Objects:** clause sets K (shared knowledge states); **Morphisms:** $f: K \rightarrow K'$ represents $K \vdash K'$ (inferential entailment); **Identity:** id_K ; **Composition:** standard.

Remark 1. The choice of a preorder category—in which any two objects are connected by at most one morphism—is intentionally conservative. It constitutes the minimal categorical structure within which universality is well-defined, and is fully adequate for our primary goal of characterizing minimal supplementation. The preorder assumption collapses all derivation paths between two knowledge states into a single binary judgment; distinguishing among different proofs of the same entailment (e.g., to capture "explanatory depth") would require a richer structure, and is deferred to future work.

Definition 2 (Interpretive Gap). An interpretive gap with respect to observation C and shared knowledge Σ is defined as the non-existence of a morphism from Σ to C in the dialogue-state category:

$$\text{Gap}(\Sigma, C) \iff \neg \exists f: \Sigma \rightarrow C$$

In dialogue terms: C is an utterance or act by the interlocutor, Σ is the current shared knowledge base, and a gap exists whenever C cannot be derived from Σ alone. Repair is the process of finding a supplementation S such that the gap is closed.

Definition 3 (Supplementation). A supplementation S for a state Σ is an object satisfying:

1. $\exists g: \Sigma \otimes S \rightarrow C$ (derivability is restored after supplementation),
2. where $\Sigma \otimes S$ is the object corresponding to
3. $\Sigma \cup S$

2.3. Minimal Supplementation

Because any gap typically permits more than one valid supplement, a principled criterion is needed to adjudicate among competing candidates. We adopt the notion of a universal property—a cornerstone of category theory—to single out the unique "smallest" supplement in the following sense:

$$\begin{array}{ccc} \Sigma \otimes S & \xrightarrow{g} & C \\ \downarrow id_{\Sigma} \otimes \phi & \nearrow & \\ \Sigma \otimes S' & & \end{array}$$

This definition formalizes “ S carries less information than any other supplementation” in terms of the existence of morphisms. Intuitively, the existence of a morphism $S \rightarrow S'$ means that the content of S is derivable from S' ; equivalently, S' is a “stronger” assumption than S .

Theorem 1 (Uniqueness of minimal supplementation). If a minimal supplementation exists, it is unique up to isomorphism.

Proof. If H_1, H_2 are both minimal, morphisms $\phi: H_1 \rightarrow H_2$ and $\psi: H_2 \rightarrow H_1$ exist. By universality, $\phi \circ \psi = id$ and $\psi \circ \phi = id$, thus $H_1 \cong H_2$.

Remark 2. Unlike information-theoretic approaches that depend on specific measures and miss structural relationships, our definition uses only morphism existence, making it more general.

2.4. Constructing Supplementation by Analogy

Supplements are rarely constructed ex nihilo; speakers typically recall a structurally analogous episode and transfer the premise that resolved it to the current context. Drawing on Abe's Analogical Abductive Reasoning (AAR) framework [6], we model this analogical transfer as a functor between dialogue-state categories, thereby preserving inferential structure across contexts:

- **Object mapping:** $F(\Sigma_{past}) = \Sigma_{current}$
- $f: \Sigma \rightarrow \Sigma'$,
- $F(f: \Sigma \rightarrow \Sigma') = F(f): F(\Sigma) \rightarrow F(\Sigma')$

Supplementation via analogy: retrieve past situation Σ_{past} with premise S_{past} , transform via F to current context ($H_{curr} = F(H_{past})$), verify $(K_{curr} \cup H_{curr}) \vdash O$.

3. Category-Theoretic Reinterpretation of Passive and Active Gaps

We reinterpret the passive and active gaps identified in our prior work [12] in category-theoretic terms.

3.1. Formalizing Passive Gaps

Passive gaps: cumulative drift where participants reach non-isomorphic states $K_A \cong K_B$ despite same dialogue sequence.

$$\Sigma_0 \xrightarrow{f_1} \Sigma_1 \xrightarrow{f_2} \dots \xrightarrow{f_n} \Sigma_n$$

$$\Sigma_n^A \cong \Sigma_n^B$$

That is, despite experiencing the same dialogue, cumulative interpretive differences lead the participants to final states that are not isomorphic.

3.2. Formalizing Active Gaps

An active gap is a creative hypothesis-formation process in which a new interpretant is intentionally generated, steering the branching of the semiotic chain. We formalize this as follows. Suppose that participant A , at state Σ , switches from the current analogical functor F to another analogical functor G :

$$F: \mathcal{D}_{past_1} \rightarrow \mathcal{D} \Rightarrow G: \mathcal{D}_{past_2} \rightarrow \mathcal{D}$$

Because of this functor switch, different supplementations may be generated even for the same observation C , e.g., $SF = F(S_{past_1})$ and $SG = G(S_{past_2})$. An active gap is an interpretive reconstruction that arises as a consequence of such a functor switch.

3.3. A Unified View

Unified view: passive gaps are object drift under a fixed analogical functor $F: \mathcal{D}_{past_1} \rightarrow \mathcal{D}$; active gaps are functor switching from F to a different functor $G: \mathcal{D}_{past_2} \rightarrow \mathcal{D}$. Both phenomena are thus expressed within a single category-theoretic formalism, differing only in whether the functor is held fixed (passive) or replaced (active). This unification supports systematic comparison of the two gap types and clarifies the structural source of their phenomenological difference.

4. Preliminary Validation

4.1. Experimental Setup

The experimental protocol involved five pairs of university students with no theatrical training, who performed approximately five minutes of unscripted improvisational acting around a shared scenario. Immediately after the performance, each participant was individually interviewed and asked to retrospectively explain the origin of each conversational move, how they had interpreted their partner's contributions, and how the

partner's behavior had appeared at that moment. Each utterance and interview response were coded for the Peircean triadic relation (sign–object–interpretant) and for inference type (abduction, induction, deduction), yielding 106 coded episodes. While our prior work [12] established the basic gap typology from this dataset, the present analysis applies the category-theoretic framework of Section 2 to the same corpus, focusing on the structural relationship between inference types and gap types that the prior set-theoretic analysis could not capture.

4.1.1. Scenario and Character Backgrounds

Each dyad received a common scenario—a walk through a park—along with individually assigned character backgrounds involving occupational stress and interpersonal difficulties. Working without scripts or rehearsal, the participants improvised freely.

4.1.2. Data Collection

Sessions were captured on video and transcribed in full. Following each performance, participants were interviewed individually. For each conversational move, three questions were posed: what motivated that particular action; how the partner's preceding contribution had been understood; and how the partner's behavior appeared at that moment.

4.1.3. Analysis Procedure

Semiotic annotation: each utterance and the corresponding interview commentary were independently coded using Peirce's triadic sign framework, assigning labels for sign, object, and interpretant. In parallel, the underlying inference type was identified as abductive (hypothesis formation), inductive (generalization from cases), or deductive (derivation from established knowledge).

Inference-target annotation: the target of each coded inference was classified along four dimensions—reasoning directed toward (1) one's own character role, (2) the partner's character role, (3) the partner as an actual person, or (4) the dyadic interaction as a whole.

Gap classification: episodes were categorized by combining inference type with inference target. Inductive episodes directed at the reasoner's own role were treated as indicators of passive gaps, reflecting gradual self-anchored drift. Abductive episodes directed at the interaction as a whole were treated as indicators of active gaps, reflecting deliberate reframing of shared relational meaning.

4.2. Results

4.2.1. Distribution of Inference Types

Annotation of the five sessions yielded 106 distinct inference episodes in total. Table 1 reports their breakdown by inference type.

Table 1 Frequency Distribution of Coded Inference Types across all Sessions

Inference type	Frequency	Percentage
Abduction	42	39.6%
Induction	58	54.7%
Deduction	6	5.7%
Total	106	100.0%

4.2.2. Association Between Inference Type and Inference Target

Cross-tabulation of inference type against inference target revealed a highly significant association ($\chi^2(6) = 32.59$, $N = 106$, $p < .01$, Cramér's $V = 0.39$). The joint distribution appears in Table 2; standardized residuals are reported in Table 3.

Table 2 Observed Frequencies by Inference Type and Inference Target

	Self	Other's Role	Other	Interaction	Total
Abduction	10	16	6	10	42
Induction	42	14	2	0	58
Deduction	1	3	1	1	6
Total	53	33	9	11	106

Analysis of standardized residuals revealed the following tendencies (Table 3).

Table 3 Standardized Residuals for the Inference-Type × Inference-Target Cross-Tabulation

Inference type	Inference-target	Std. Residual
Abduction	Interaction	+2.70
Abduction	Self's role	-2.40
Induction	Self's role	+2.41
Induction	Interaction	-2.45

4.2.3. Testing the Category-Theoretic Predictions

Two cells exceeded the conventional threshold of $|\text{residual}| \geq 1.96$ (corresponding to $p \leq .05$): abduction paired with interaction-level reasoning (residual = +2.70), and induction paired with self-role reasoning (residual = +2.41). Every cell involving deduction produced residuals below this threshold, consistent with the theoretical prediction that deduction does not drive gap formation.

4.3. Case Analysis

4.3.1. Example 1: Repair via Empathy: Resolving a Passive Gap (Pair 3)

Utterance sequence:

B: "It's like things aren't going well with the client... their requests and all that."

A: "The client's requests are kind of unclear—more emotion-based..."

B: "But if you're not someone who's genuinely trying, you don't even get to worries at that level of resolution, right?"

Interview excerpt:

B: "They increased the 'resolution' of the situation the person is in regarding the earlier worry. Once you understand that, you worry... and I felt like maybe it's not something to worry about, or rather, I saw it differently."

Category-theoretic analysis:

- **Supplementation H:** "You worry because you're trying seriously." **Minimality:** minimal logical step deriving "serious engagement" from "worry." These alternatives do not provide a structurally simpler route to the observation; therefore, within the present qualitative analysis, H is treated as the minimal supplement. **Existence of morphisms:** from alternative explanations such as "lack of ability" or "lack of experience," one cannot derive *S* (i.e., no morphism exists), hence *S* is *more minimal*.

4.3.2. Example 2: Topic Shift and Passive Gap (Pair 1)

The following episode illustrates how a passive gap arises through cumulative topic drift without either participant intending a divergence.

Utterance sequence:

B: "Work has been really rough lately—nothing's going well."

A: "Yeah, I know that feeling. Actually, that's exactly why I started doing yoga recently."

B: (puzzled) "Yoga? That came out of nowhere."

Post-performance interview:

B: "I thought the conversation would open up toward work troubles—I was preparing to empathize. Then yoga just appeared. It completely caught me off guard."

Category-theoretic analysis:

Both participants began from a shared state Σ_0 focused on professional difficulty. A applied an inductive inference linking "stress relief" to "yoga," updating her state to Σ_A ("yoga as coping"). B applied an inductive inference anchored to "continued discussion of work," updating his state to Σ_B ("more work detail forthcoming"). Because both updates proceeded under the same analogical functor—drawing on each participant's respective prior experiences—but with different inductive generalizations, the resulting states $\Sigma_A \not\cong \Sigma_B$ constitute a passive gap. The drift was unintentional and accumulated across two turns rather than arising from a single deliberate move. This is precisely the pattern predicted by the object-drift formalization: same functor, diverging objects.

4.3.3. Example 3: Functor Switching and Active Gap (Pair 5)

This episode illustrates an active gap arising from an explicit functor switch, where one participant deliberately reframes the interaction using a new interpretive schema.

Utterance sequence:

A: "A fresh perspective really matters—why don't we do yoga together?"

B: "Yoga? Wait, did you start yoga because things have been difficult lately?"

Post-performance interview:

A: "My thought was that trying something new opens up new angles on things."

B: "She said work was going badly, then yoga just entered the picture. I needed to understand why."

Category-theoretic analysis:

At the moment of A's utterance, B was operating under functor $F: D_{\text{past1}} \rightarrow D$ ("problem-sharing $\rightarrow$ empathize" frame). A's proposal invoked a different functor $G: D_{\text{past2}} \rightarrow D$ ("problem-solving $\rightarrow$ propose new activity" frame), generating supplement $S_G = G(S_{\text{past2}})$ = "new activity yields new perspective." B's question ("did you start yoga because...") is a repair initiation that acknowledges the gap created by this functor switch and seeks to understand the new frame. A's explanation—making the "fresh perspective" reasoning explicit—provides the minimal supplement needed for B to adopt functor G . Once B accepted S_G as the appropriate supplement, the active gap was resolved and the two participants continued under a shared functor G . This episode exemplifies the active gap mechanism: the gap arises not from gradual drift but from a single deliberate functor-level shift.

5. Discussion**5.1. Theoretical Contributions**

This paper makes two distinct types of contribution, and their relationship deserves explicit clarification. The first is a formal contribution: we introduce a category-theoretic framework for dialogue repair and prove that a minimal supplement, when it exists, is unique up to isomorphism (Theorem 1). This result is a genuine mathematical consequence of the universal property and cannot be derived from set-theoretic or information-theoretic formulations, which lack the structural apparatus needed to guarantee such uniqueness. The second contribution is empirical: we report a preliminary coding study ($N = 106$ episodes, five dyads) showing systematic associations between inference types and gap types ($\chi^2(6) = 32.59$, $p < .01$, Cramér's $V = 0.39$). These results are consistent with the framework's predictions, but they do not constitute a formal test of the categorical structure itself. The same statistical pattern could in principle be predicted by any theory that distinguishes cumulative-drift

mechanisms from deliberate-reframing mechanisms, without invoking category theory. The empirical results should therefore be read as establishing plausibility and motivating further investigation, not as validating the categorical formalism.

Comparison with traditional approaches. Three mainstream formalizations of "minimal supplementation" exist in the literature, each with structural limitations that the present framework overcomes. (i) Set-theoretic approaches define minimality as smallest cardinality, but cardinality ignores the logical entailment relations among candidates—two supplements of equal size may differ substantially in inferential strength. (ii) Information-theoretic approaches (e.g., MDL, entropy minimization) quantify minimality as a scalar, but the result is measure-dependent and cannot express whether one supplement is structurally derivable from another, nor model the compositionality of sequential repairs. (iii) Logical abduction models, including Abe's AAR framework [6], rigorously characterize hypothesis formation but provide no universal criterion that singles out one supplement as uniquely minimal; multiple equally plausible hypotheses may coexist without a principled selection mechanism. The present framework replaces these scalar or cardinality-based notions with a morphism-existence criterion derived from the universal property of category theory [7, 8]. This yields three concrete advantages absent from the alternatives: a provable uniqueness guarantee (Theorem 1), native compositionality via morphism composition, and a single formalism that subsumes both passive and active gap types.

Formal precision: the framework characterises minimality as a structural property—the existence of a unique factoring morphism through which every other valid supplement passes—rather than as a numerical quantity to be minimised. This makes the notion of "most economical supplement" independent of any particular information measure or cardinality criterion, and therefore more general.

Category-theory-specific result: Theorem 1 is the claim in this paper that is most distinctively categorical. It establishes that the universal property singles out a unique (up to isomorphism) minimal supplement—a guarantee that has no counterpart in approaches based on cardinality minimisation or entropy, where the "minimal" element depends on the chosen measure and need not be structurally unique. This uniqueness result is, arguably, the clearest demonstration of the added value that the categorical machinery brings.

Compositionality: morphism composition provides a natural account of sequential supplementation, where one repair lays the inferential groundwork for the next. This is especially relevant for multi-turn repair sequences, which are common in natural dialogue but are difficult to model in frameworks that treat each supplement as an isolated act.

Unified treatment: passive gaps (object drift under a fixed analogical functor) and active gaps (functor switching) are expressed within a single categorical

formalism. This makes the structural difference between the two gap types precise and supports systematic comparison, whereas treating them as merely qualitatively distinct phenomena would not yield the same formal clarity.

Taken together, these findings support the view that category theory provides a particularly apt mathematical language for dialogue repair. Unlike set-theoretic minimization—which reduces "minimal" to "smallest in cardinality" and thereby loses the logical structure among candidates—our approach characterizes minimality through morphism existence: a supplement is minimal if every other valid supplement is reachable from it via a morphism ($H_{\min} \rightarrow H$, i.e., $H_{\min} \vdash H$), meaning the minimal supplement implies (derives) every other valid supplement—not the reverse. This structural conception aligns naturally with speakers' intuitions: the "best" explanation for a partner's unexpected move is not merely the shortest one, but the one from which all other reasonable explanations follow as logical consequences.

The three case studies presented in Section 4 illustrate this concretely. In Example 1 (Pair 3), the supplement "you worry because you are genuinely engaged" is minimal in the technical sense: no simpler assumption suffices to derive the observation, and the alternative explanations (lack of skill, lack of experience) do not admit a morphism from S , confirming S 's minimality. In Example 3 (Pair 5), minimality applies at the level of the functor switch itself: the simplest reframing that accounts for the yoga proposal is the "new-activity" frame, not any more specific frame about yoga in particular.

5.2. Correspondence with Empirical Findings

Induction $\times$ self-role (standardized residual = +2.41): the positive association between inductive inference and reasoning about one's own character role supports the passive-gap mechanism, in which a participant's self-representation drifts gradually through self-anchored generalisation. The categorical counterpart is object drift under a fixed analogical functor. It should be noted, however, that this association is consistent with the framework rather than uniquely explained by it: a theory that simply posits "induction drives self-role reasoning" would produce the same statistical result without invoking category theory.

Abduction $\times$ interaction (standardized residual = +2.70): the positive association between abductive inference and interaction-level reasoning supports the active-gap mechanism, in which a participant introduces a new interpretive frame through deliberate hypothesis formation. The categorical counterpart is functor switching. As with the induction result, the same association could in principle be predicted by a theory that identifies abduction with global reframing without the categorical apparatus; the formalization's contribution is precision and formal tractability, not empirical exclusivity.

Deduction showed no significant associations with any inference target (all residuals below ± 1.2). This null result

is consistent with the prediction that deduction, as morphism composition within an existing functor, does not generate supplementation and thus does not drive gap formation. While the result aligns with the theory, a null result alone cannot rule out alternative explanations for the absence of association, and should be interpreted with appropriate caution.

5.3. Limitations and Future Work

Three sets of limitations deserve explicit acknowledgment. First, and most importantly, the relationship between the categorical formalization and the empirical evidence is indirect. The statistical results establish that inference types and gap types co-vary systematically, but they do not directly test the category-theoretic structure. The framework's distinctive contribution—the universal characterisation of minimality and the uniqueness result of Theorem 1—is a formal result that has not been operationalised or empirically tested in the present work. A meaningful empirical test would require identifying predictions that follow from the categorical structure but not from weaker theories, and designing studies capable of discriminating between them. Second, the case analyses in Section 4.3 are necessarily post hoc: the categorical interpretation is assigned after observing the data, and no guarantee exists that it is the uniquely correct interpretation. In particular, the minimality claims—for example, that no morphism exists from an alternative explanation to the identified supplement S —rest on qualitative judgement rather than formal proof, because the full clause-set semantics of each episode are not formally specified.

Third, the sample size (five pairs, 106 episodes) limits statistical power and generalisability; the moderate effect size (Cramér's $V = 0.39$) should be treated as exploratory pending replication with larger samples.

To address these limitations, several concrete next steps are envisaged. First, the indirectness of empirical validation can be addressed by designing experiments that yield predictions derivable from the categorical structure but not from weaker theories—for example, predictions about the compositional behavior of sequential repairs (which morphism composition uniquely captures) or about the structural relations among competing supplements within a single episode. Second, the qualitative nature of the minimality judgments in Section 4.3 can be mitigated by introducing independent multi-coder annotation: having multiple coders apply the categorical interpretation to the same episodes, with inter-rater reliability measured for the supplement-identification and morphism-existence judgments, would provide a more objective basis for the case-analytic claims. Third, generalizability can be strengthened by replicating the study with a larger and more diverse sample, including dyads from different cultural and professional backgrounds, to assess the robustness of the gap-type typology across populations.

A further limitation concerns the existence of minimal supplementations. The present paper proves uniqueness

(Theorem 1) but takes existence for granted. In principle, a supplementation lattice could have no minimum element—for example, if there is an infinite descending chain of increasingly weak supplements. Characterizing the conditions under which a minimal supplement is guaranteed to exist (e.g., well-foundedness conditions on the lattice of clause sets) is a necessary step toward making the framework computationally tractable.

Finally, the current framework treats the dialogue-state category as a preorder, meaning that all proofs of an entailment are identified. A richer treatment—for instance, one that takes morphisms to be proof terms up to a suitable equivalence relation—would allow distinctions between supplements that reach the same conclusion via different inferential paths, potentially capturing intuitions about explanatory depth or cognitive effort that the preorder model currently elides.

6. Conclusion

This study formalized dialogue repair via category-theoretic universality, unifying passive/active gaps as object drift and functor switching. Experimental validation (10 participants, 106 episodes) confirmed significant inference-gap associations ($\chi^2(6)=32.59$, $p<.01$, $V=0.39$), supporting the framework's plausibility.

Beyond the specific results, this paper illustrates a broader methodological point: category theory's emphasis on morphisms and universal properties—rather than on the internal structure of objects—makes it well suited to modeling communicative phenomena where the relationship among interpretations matters more than any particular interpretation's intrinsic content. The shift from set-theoretic to category-theoretic thinking aligns with a move from "what is in the supplement" to "how supplements relate to one another," which we argue is the epistemically relevant question in dialogue repair.

Three directions stand out for future work. First, scaling the empirical validation to at least twenty pairs will increase statistical power and allow examination of individual and cultural differences in gap type and repair strategy. Second, developing computational algorithms for minimal-supplement search—drawing on results from lattice theory and automated abduction—will be necessary to apply the framework in real-time dialogue systems, including dialogue agents based on large language models, where grounding gaps have recently been documented [14]. Third, extending the base category from a preorder to a richer structure (such as a bicategory of contexts, or a category enriched over a suitable monoidal structure) will allow the framework to capture proof-theoretic distinctions and compositionality at the level of inferential strategies, not just conclusions. Together, these extensions aim toward a computational theory of human dialogue reasoning that is both formally precise and empirically grounded.

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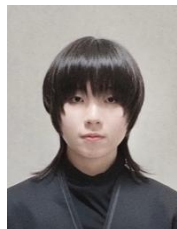
Dr. Akinori Abe



He received M.E. (1988) and PhD (1991) from the University of Tokyo. His research interests include abduction, analogical reasoning, data mining, Chance Discovery, curation, and language sense processing. He worked at NTT Communication Science Laboratories (1991–2000) and has been a professor at Chiba University since 2012.

Authors Introduction

Ms. Yukiko Furuya



She is a PhD student in the Graduate School of Informatics at Chiba University, with training in design science (BA), cognitive science (MA), and data science (PhD). Her research bridges empirical analyses of improvisational acting and conversational data with semiotics and category theory, focusing on abduction and the systematicity of reasoning in dialogue, narrative, and human–AI interaction.