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Hierarchy Based on Neighborhood Template about *k*-Neighborhood Template $\mathcal{A}$-Type Three-Dimensional Bounded Cellular Acceptor

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Abstract

Recently, due to the advance in dynamic image processing, computer animation, augmented reality (*AR*), and so forth, it has become increasingly apparent that the study of four-dimensional pattern processing (three-dimensional pattern processing with time axis) should be very important. Thus, the study of four-dimensional automata as the computational model of four-dimensional pattern processing has been meaningful. From this point of view, we first introduced a four-dimensional automaton in 2002. In the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low-dimensional space. In this paper, from this viewpoint, we introduce a new computational model, *k-neighborhood template* $\mathcal{A}$*-type three-dimensional bounded cellular acceptor* (abbreviated as $\mathcal{A}$-$3BCA(k)$) on four-dimensional input tapes, and discuss hierarchy based on neighborhood template about $\mathcal{A}$-$3BCA(k)$.

Keywords: cellular acceptor, computational complexity, configuration-reader, converter, four-dimension, neighbor

1. Introduction and Preliminaries

Due to the advances in many application areas such as computer animation, dynamic image processing, and so on, the study of four-dimensional pattern processing has been of crucial importance. Thus, the study of four dimensional automata as the computational models of four-dimensional pattern processing has been meaningful. From this point of view, we first proposed four-dimensional automata as computational models of four-dimensional pattern processing in 2002 [4], and investigated their several accepting powers. By the way, in the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low dimensional space. So, from this viewpoint, we introduce a new computational model, *k*-neighborhood template $\mathcal{A}$-type three-dimensional bounded cellular acceptor (abbreviated as $\mathcal{A}$-$3BCA(k)$) on four-dimensional tapes in this paper, and discuss some basic properties. An $\mathcal{A}$-$3BCA(k)$ consists of a pair of a converter and a configuration-reader. The former converts the given four-dimensional tape to three-dimensional configuration. The latter determines whether or not the derived three-dimensional configuration is accepted, and

concludes the acceptance or non-acceptance of given four-dimensional tape(see Fig.1). When an input four-dimensional tape is presented to the $\mathcal{A}$ -3*BCA*(*k*), a three-dimensional cellular automaton as the converter first reads it to the future direction at unit speed (i.e., one three-dimensional rectangular array per unit time). From this process, the four-dimensional tape is converted to a configuration of the converter which is a state matrix of a three-dimensional cellular automaton. Second, three-dimensional automaton as the configuration-reader reads the configuration and determines its acceptance. We say that an input remarkable pixel or voxel in neighbor. In other words, we deal with 5- and 9-connectedness in the two-dimensional case, and 7- and 27-connectedness in the three-dimensional case in this paper. In Addition, in our another paper, we mainly investigate how the difference of configuration-reader affects the accepting powers of $\mathcal{A}$ -3*DBCA*(*k*)'s. When an input four-dimensional tape is presented to the $\mathcal{A}$ -3*BCA*(*k*), a three-dimensional cellular automaton as the converter first reads it to the future direction at unit speed (i.e.,one three-dimensional rectangular array per unit time), and a two-dimensional cellular automaton as the converter next reads a converted three-dimensional configuration downward at unit speed (i.e., one plane per unit time). From this process, the four-dimensional tape is converted to a configuration of the converter which is a state matrix of a two-dimensional cellular automaton. Second, two-dimensional automaton as the configuration-reader, reads the configuration and determines its acceptance. We say that an input four-dimensional tape is accepted by the $\mathcal{A}$-3*BCA*(*k*) if and only if the configuration is accepted by the configuration-reader. Therefore, the accepting power of the $\mathcal{A}$-3*BCA*(*k*) depends on how to combine the converter and the configuration-reader. An $\mathcal{A}$-3*DBCA*(*k*) ($\mathcal{A}$-3*NBCA*(*k*)) is called a *k*-neighborhood template $\mathcal{A}$-type three-dimensional deterministic bounded cellular acceptor (*k*-neighborhood template $\mathcal{A}$-type three-dimensional nondeterministic bounded cellular acceptor). A *DA*[1] (*NA*, *DB*[5], *NB*, *DO*[2], *NO*, *DOP*[3], *NOP*, *DP*[3], *NP*, *DTM*[4], *NTM*) is called four-dimensional tape is accepted by the $\mathcal{A}$ -3*BCA*(*k*) if and only if the configuration is accepted by the configuration-reader. Therefore, the accepting power of the $\mathcal{A}$ -3*BCA*(*k*) depends on how to combine the converter and the configuration-reader. An $\mathcal{A}$ -3*DBCA*(*k*) ($\mathcal{A}$ -3*NBCA*(*k*)) is called a *k*-neighborhood template $\mathcal{A}$-type three-dimensional deterministic bounded cellular acceptor (*k*-neighborhood template $\mathcal{A}$-type three-dimensional nondeterministic bounded cellular acceptor). This paper mainly investigates how the difference of the neighborhood template of the converter affects the accepting powers of $\mathcal{A}$ -3*DBCA*(*k*)'s. In general, it is well known that two-dimensional digital pictures have 4- and 8-connectedness, and three-dimensional digital pictures have 6- and 26-connectedness. However, we include the a three-dimensional deterministic finite automaton (three-dimensional nondeterministic finite automaton, deterministic three-dimensional bounded cellular acceptor, nondeterministic three-dimensional bounded cellular acceptor, three-dimensional deterministic on-line tessellation acceptor, three-dimensional nondeterministic on-line tessellation acceptor, deterministic three-way parallel/sequential array acceptor, nondeterministic three-way parallel/sequential array acceptor, deterministic four-way parallel/sequential array acceptor, nondeterministic four-way parallel/sequential array acceptor, three-dimensional deterministic Turing machine, three-dimensional nondeterministic Turing machine). Let *T*(*M*) be the set of four-dimensional tapes accepted by a machine *M*, and let $\mathcal{L}$[$\mathcal{A}$-3*DBCA*(*k*)] = {*T*|=*T*(*M*) for some $\mathcal{A}$-3*DBCA*(*k*) *M*}. $\mathcal{L}$[$\mathcal{A}$-3*NBCA*(*k*)], etc. are defined in the same way as $\mathcal{L}$[$\mathcal{A}$-3*DBCA*(*k*)].

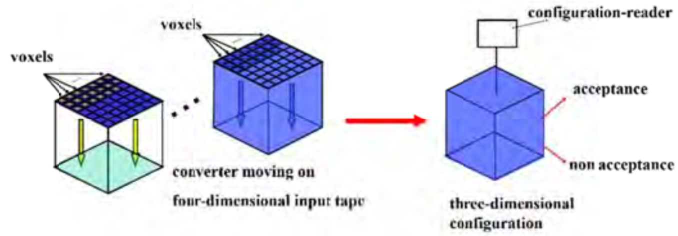

Fig. 1:Four-dimensional Input Tape.

Let Σ be a finite set of symbols. A *four-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ. The set of all four-dimensional tapes

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over Σ is denoted by $\Sigma^{(4)}$. Given a tape $x \in \Sigma^{(4)}$, for each integer $j(1 \leq j \leq 4)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\Sigma^{(n_1,n_2,n_3,n_4)}$. $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j(1 \leq j \leq 4)$, as the four-dimensional input tape y satisfying the follwing conditions:

(i) for each $j(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$;

(ii) for each $r_1, r_2, r_3, r_4 (1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$, $1 \leq r_4 \leq l_4(y))$, $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 \leq 1, r_2 + i_2 \leq 1, r_3 + i_3 \leq 1, r_4 + i_4 - 1)$. (We call $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ the $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$*-segment of x.)*

We let each sidelength of each input tape of these automata be equivalent in order to increase the theoretical interest.

2. Main Result

This section investigates how the difference of the neighborhood template of converter affects the accepting powers of $\mathcal{A}$-3*BCA*(k)'s. First, we investigate the difference between the accepting powers of one-neighbor and seven-neighbor.

Lemma 1. *Let* $T_1 = \{x \in \{0,1,2\}^{(4)} | \exists n \geq 1\ [l_1(x) = l_2(x) = l_3(x) = l_4(x) = n\ \&\ \forall i(1 \leq i \leq n)[x[(1,1,n,i), (n,n,n,i)] = x[(1,1,n,i), (n,n,i,n)]]]\}$. *Then,* (1) $T_1 \in \mathcal{L}[DA\text{-}3DBCA(7)] \cap \mathcal{L}[DB\text{-}3DBCA(7)] \cap \mathcal{L}[DO\text{-}3DBCA(7)]$, and (2) $T_1 \notin \mathcal{L}[TM\text{-}3NBCA(1)]$.

Proof: (1) The proof is omitted here since it is easy to prove. (If necessary, see the proof of Lemma 1(1) in [6].) (2) We can show $T_1 \notin \mathcal{L}[TM\text{-}3NBCA(1)]$ by using the same technique as in the proof of theorem 1 in [7]. Suppose that there exists a *TM*-3*NBC*A(1) $M = (R,B)$ accepting T_1, where R is a converter and B is a configuration-reader.

Let S be the number of states of each cell of R. For each $n \leq 1$, let

$V(n) = \{x \in \{0,1\}^{(4)} | l_1(x) = l_2(x) = l_3(x) = l_4(x) = n + 1\ \&\ \forall i(1 \leq i \leq n) x[(1,1,1,i),(n+1,n+1,n,i) \in \{0\}^{(3)}$, $x[(1,1,n+1,n+1), (n+1,n+1, n+1, n+1)] \in \{0\}^{(3)}\}$ and $W(n) = V(n) \cap T_1$.

When $1 \leq i_j \leq l_j(x)$ for each $j(1 \leq j \leq 4)$, let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4). Furthermore, we define

Also, for each $x \in V(n)$ and for each $i(1 \leq i \leq n)$, let $\rho(x) \equiv$ the configuration of R just after reading x, $\rho_U(x) \equiv [(1,1,n+1,i),(n+1,n+1,n+1,i)]$*-segment* of $\rho(x)$, and $\rho_D(x) \equiv$ the $[(1,1,n+1,i),(n+1,n+1,n+1,i)]$*-segment* of $\rho(x)$. Further, for each n ≥ 1, let $C(n) = \{\rho_D(x) | x \in W(n)\}$. Then, the following two propositions must hold.

Proposition 1. *For each* $i(1 \leq i \leq n)$,

(i) *For any two tapes x, y* $\in V(n)$ *such that their* $[(1,1,1,i),(n+1,n+1,n,i)]$*-segment are identical,* $\rho_U(x) = \rho_U(y)$,

(ii) *For any two tapes x, y* $\in V(n)$ *such that their* $[(1,1,1,i),(n+1,n+1,n,i)]$*-segments are identical,* $\rho_D(x) = \rho_D(y)$.

[**proof:** Since R is deterministic and is of one-neighbor, the proof is easy to see. □]

Proposition 2. *For any two different tapes x, y* $\in W(n)$, $\rho_D(x) \neq \rho_D(y)$.

[**Proof:** Suppose, on the contrary, that $\rho_D(x) = \rho_D(y)$. Consider the tapes $z \in V(n)$ satisfying the following two conditions for each $i(1 \leq i \leq n)$:

① $z[(1,1,1,i),(n+1,n+1,n,i)] = x[(1,1,1,i),(n+1,n+1,n,i)]$,

② $z[(1,1,n+1,i), (n+1,n+1,n+1,i)] = y[(1,1,n+1,i),(n+1,n+1,n+1,i)]$.

Clearly, $x \in W(n) \subseteq T_1$. Thus x is accepted by M. Therefore, $\rho(x)$ is accepted by B.

On the other hand, it follows from① and Proposition 1 (i) that $\rho_U(x) = \rho_U(y)$, and it follows from ② and Proposition 1 (ii) that $\rho_D(x) = \rho_D(y)$. Further, from the foregoing assumption $\rho_D(z) = \rho_D(y)$, it follows that $\rho_D(z) = \rho_D(x)$. From $\rho_U(z) = \rho_U(x)$ and $\rho_D(z) = \rho_D(x)$, it follows that $\rho(z) = \rho(x)$.

Since $\rho(x)$ is accepted by B, $\rho(z)$ is also accepted by B. Consequently, z is also accepted by M. This is a contradiction.(Note that $z \notin T_1$.) □]

Proof of Lemma 1(*continued*) : As is early seen, $|W(n)| = 2^{n(n+1)(n+1)}$ and $|C(n)| \leq s^{(n+1)(n+1)}$

Therefore, it follows for large n that $|W(n)| < |C(n)|$. Consequently, it follows for such large n that there must be two different tapes $x,y \in W(n)$ such that $\rho_D(x) \neq \rho_D(y)$. This contradicts Proposition 2. □

Theorem 1. *For each* $\mathcal{A} \in$ *{DA, NA, DB, NB, DO, NO, DOP,NOP, DP, NP, TM} and for each* $X \in \{D, N\}$, $\mathcal{L}[\mathcal{A}\text{-}3XBCA(1)] \subsetneq \mathcal{L}[\mathcal{A}\text{-}3XBCA(7)]$.

Proof: The inclusion relation holds immediately definitions. Further, it is easily seen from Lemma 1 and Proposition 1 in [1] that the theorem holds. □

We investigate the difference between the accepting powers of seven-neighborhood and twenty-seven-neighbor. As shown later in Theorem 2 and 3, different situations emerge depending on whether the converter is deterministic or nondeterministic. First, we consider the case when the converter is deterministic.

Theorem 2. *For each* $\mathcal{A} \in$ *{ DA, NA, DB, NB, DO, NO }*, $\mathcal{L}[\mathcal{A}\text{-}3DBCA(7)] \subsetneq \mathcal{L}[\mathcal{A}\text{-}3DBCA(27)]$.

Proof : The inclusion relation holds immediately from definitions. Further, it is easily seen from Theorem 3 and Proposition 1 in [7] that the theorem holds. □

We conclude this section by investigating the difference between the accepting powers of seven-neighbor and twenty-seven-neighbor for the case when converter is nondeterministic.

Theorem 3. *For each* $\mathcal{A} \in$ *{ DA, NA, DB, NB, DO, NO, DOP, NOP, DP, NP, TM }*, $\mathcal{L}[\mathcal{A}\text{-}3DBCA(7)] = \mathcal{L}[\mathcal{A}\text{-}3NBCA(27)]$.

Proof: For each $\mathcal{A} \in$ { *DA, NA, DB, NB, DO, NO, DOP, NOP, DP, NP, TM* }, it is obvious that $\mathcal{L}[\mathcal{A}\text{-}3NBCA(7)] \subseteq \mathcal{L}[\mathcal{A}\text{-}3NBCA(27)]$ from definitions. Below, we show that $\mathcal{L}[\mathcal{A}\text{-}3NBCA(7)] \supseteq \mathcal{L}[\mathcal{A}\text{-}3NBCA(27)]$. Given an $\mathcal{A} \in$ { *DA, NA, DB, NB, DO, NO, DOP, NOP, DP, NP, TM* }, we let $M = (R, B)$ be an $\mathcal{A}$-3*NBCA*(27). We now consider the $\mathcal{A}$-3*NBCA*(7) $M' = (R', B'$) which acts as follows.

Suppose that a four-dimensional input tape x with each sidelength n ($n \geq 2$) is presented to M. Since R' has seven neighbors, each voxel of R' can refer directly to the states of , remarkable, north, south, east, west, up, and down neighbor voxels, but not other neighbor voxels. Therefore, by guessing the seven states of neighbor voxels which cannot be referred to directly, and by checking whether or not this guess is convect, each voxel of R' simulates the action of the corresponding voxel of R. In fact, $M' = (R', B')$ acts as follows.

(i) Action of the converter R'

R' starts to act with the same initial configuration with one of R. That is, every voxel reading the boundary symbol is in $q_{\#}$, which is the boundary state of R, and all of the other voxels are in q_0, which is the initial state of R. Next, the seven states of neighbor voxels of R is easily simulated by R', and the twenty states of other neighbor voxels of R are guessed nondeterministically by R.

(ii) Action of the configuration-reader B'

B' accepts the configuration of R' just after reading x (say, $\rho(x)$), if and only if the following ①,②,③ are satisfied.

① For each i, j, k ($1 \leq i, j, k \leq n$), no (i, j, k)-voxel enters the dead state.

② The information guessed by any (i, j, k)-voxel on the last three-dimensional rectangular array of four-dimensional input tape is correct.

③ Let h be a mapping extracting one state, which is obtained by simulating the action of the corresponding voxel of R, from the states stored in the state of (i, j, k)-voxel. Let h be a projection which is obtained by extending the mapping h. Then $h(\rho(x))$ is accepted by B.

In ① and ② in the foregoing, check whether or not R' can correctly simulate the action of R. Therefore, it is clear thet $h(\rho(x))$ reflects the configuration of R just after reading x, if ① and ② are satisfying. It is easily seen that $T(M') = T(M)$ for $M = (R', B')$. Thus, $\mathcal{L}[\mathcal{A}\text{-}3NBCA(7)] \supseteq \mathcal{L}[\mathcal{A}\text{-}3NBCA(27)]$. This completes the proof of the theorem. □

3. Conclusions

In this paper, we investigated how the difference of neighborhood template of the converter affects the accepting powers of *k*-neighborhood template $\mathcal{A}$-type three-dimensional bounded cellular acceptor (abbreviated as $\mathcal{A}$-3*BCA*(*k*)). Generally speaking, when the converter is deterministic, the accepting power of the $\mathcal{A}$-3*BCA*(*k*) tends to be more powerful as the number of neighborhood cells of the converter increases or the accepting power of the configuration-reader is more powerful. However, this tendency is not always true when the converter is nondeterministic.

We conclude this paper by giving two open problems.

(1) For each $\mathcal{A} \in \{DOP, NOP, DP, NP, DTM, NTM\}$, $\mathcal{L}[\mathcal{A}\text{-}3DBCA(7)] \subsetneq \mathcal{L}[\mathcal{A}\text{-}3DBCA(27)]$?

(2) $\mathcal{L}[NO\text{-}3NBCA(7)] \subsetneq \mathcal{L}[DOP\text{-}3NBCA(27)]$?

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