

## Adaptive Sliding Mode Control for Magnetic Levitation Vehicles \*

Juanjuan He and Yingmin Jia

The Seventh Research Division, Beihang University (BUAA),  
Beijing, 100191, China

E-mail: [hejuanjuan@smss.buaa.edu.cn](mailto:hejuanjuan@smss.buaa.edu.cn); [ymjia@buaa.edu.cn](mailto:ymjia@buaa.edu.cn)

### Abstract

This paper focuses on stability control for the levitated positioning of the magnetic levitation vehicle system. For the nonlinear magnetic levitation system model, the output feedback linearization method is employed to derive a global linearization error model. However, there exists uncertain item in the error model. To stabilize this error model, the adaptive sliding mode control method is used here. Simulations show that the magnetic levitation system can be stability quickly under controlled by the proposed control scheme.

*Keywords:* Magnetic levitation vehicle system; nonlinear model; output feedback linearization; adaptive sliding mode control

### 1. Introduction

As the smart transportation establishing becomes a hot research issue in recent years, how to improve the operating efficiency and convenience of the entire city is becoming increasingly important. Therefore, it is necessary to establish a new high-speed and efficient means of transport. Up to 500km/h of speed, the maglev train can greatly improve the transport capability. Furthermore, magnetic levitation technology has seen a rapid development since the 20th century, and made the interesting in the field of real-life applications [1], such as transportation systems [2,3], wind tunnel levitation[4], magnetic bearing systems and antivibration table. However, the design of maglev system controller still presents plenty of formidable challenges [5] including the nonlinear solenoid model, instability, and inevitably uncertain parameters. Hence, a proper controlling mode

should be designed for the suspension system to guarantee the robust stability of the entire system. For traditional methods, nonlinear magnetic levitation vehicle system model is linearized at one equilibrium point [6], which leads to the fact that the design of the controller seriously depends on the selected equilibrium point. So this type of linearization cannot be applied to the situation that the gap is a large range of variation. Thus if the traditional linearization method is applied to this situation, it may lead to the performance degradation or instability of the system. Hence, a new solution should be proposed.

In this paper, latest control method is applied in allusion to the magnetic levitation planner. Firstly, output feedback linearization technique is applied to the nonlinear magnetic levitation vehicle system model, which derives a global linearization error model. However, the simplified model is unstable and includes uncertain factors, so the adaptive sliding mode

---

\* The Seventh Research Division, Beihang University,  
No.37, Xueyuan Road, Haidian Dist., Beijing 100191, China.

controller design method is used here, which makes the linearization error system stable. Finally, a series of simulation experiments are shown in this paper, which is used to demonstrate the effects of output feedback linearization method and adaptive sliding mode controller.

This paper is organized in five sections. In the next section, model analysis of magnetic levitation vehicle system is formulated, and output feedback linearization is proposed. Section 3 is related to the controller design. Simulation results are presented in section 4. Finally, in section 5, some concluding remarks are outlined.

## 2. Model Analysis of The Maglev System

### 2.1. Nonlinear Solenoid Model Analysis

The schematic diagram of the magnetic levitation system is depicted in Fig.1. For the sake of achieving the goal of high-precision positioning, a complex magnetic levitation system vehicle model needs to be analyzed thoroughly.

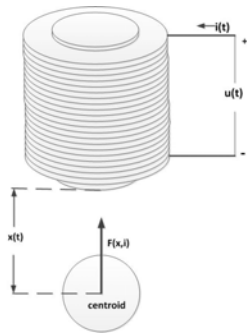


Fig.1. Schematic diagram of magnetic levitation system

Before deriving, we make the reasonable assumption that the air-gap flux leakage is zero. Then, the idealized nonlinear solenoid model can be calculated as

$$F = \frac{\mu_0 n^2 \sigma i^2(t)}{4x^2(t)} \quad (1)$$

where  $F$  is the electromagnetic force between the coil and suspension; the constant  $\mu_0 = 4\pi \times 10^{-7} \text{H/m}$  is the absolute magnetic permeability;  $n$  is the coil turn;  $\sigma$  is the magnetic cross section;  $i(t)$  is the coil current, and  $x(t)$  is the levitation height between coil and suspension. The coil model is given by

$$u(t) = Ri(t) + \frac{d\psi(x,i)}{dt} \quad (2)$$

where  $u$  is control voltage, and  $R, \Psi$  are circuitous resistance and flux linkage respectively.

### 2.2. System Dynamics

The complete model of the maglev system can be derived by using *Newton's Law* as follow

$$\sum \bar{F} = m\ddot{x}(t) \quad (3)$$

Firstly, to obtain the system dynamics, we apply Eq.(1) and Eq.(2) to Eq.(3). As a result, the dynamic equations can be rewritten as

$$m\ddot{x}(t) = \frac{\mu_0 n^2 \sigma i^2}{4x(t)^2} - mg \quad (4)$$

where  $m$  is mass of the suspension;  $g$  is acceleration of gravity; and  $\sigma$  is magnetic area .

### 2.3. Analysis of Nonlinear Model

Let the states be chosen as  $x_1 = x, x_2 = \dot{x}, x_3 = i$ .  $u$  is the input voltage of the system changing from 0 to 5 volts, and  $X = [x_1 \ x_2 \ x_3]^T$  is the state vector. Thus, the state-space model of the magnetic levitation system can be written as

$$\begin{aligned} \dot{X} &= F(X) + G(X)U(t) \\ Y(t) &= x_1 \end{aligned} \quad (5)$$

where  $S = \frac{\mu_0 \sigma n^2}{2}$ , and  $F(X), G(X), U(t)$  are

$$F(X) = \begin{bmatrix} x_2 \\ \frac{S}{2m} \left( \frac{x_3}{x_1} \right)^2 - g \\ \frac{S}{2} \frac{x_2 x_3}{x_1} - R x_1 x_3 \end{bmatrix}, G(X) = \begin{bmatrix} 0 \\ 0 \\ x_1 \end{bmatrix}, U(t) = u(t).$$

### 2.4. Linearization of System Model

For the purpose of the afterward controller design, the feedback linearization is applied to the system model shown as Eq.(5) in this subsection. As a result, the third derivative of system outputs can be finally obtained as follows

$$\ddot{Y}=B(C+KU) \tag{6}$$

where  $K=\frac{x_3}{x_1}$ ,  $B=\frac{1}{m}$ , and  $C=-\frac{Rx_3^2}{x_1}-\frac{Sx_2x_3^2}{2x_1^3}$ .

Obviously, when  $x_1, x_3 \neq 0$ , then  $K \neq 0$ . In physics system,  $x_j, j \in (1,3)$ , which are Central levitation height of the suspension and coil current respectively, thus their product could not be zero, so the decoupling matrix  $K$  is invertible.

Thus, the decoupling control inputs can be obtained as

$$U=K^{-1}(U_c-C) \tag{7}$$

where  $U_c=v$ .

Then the linearized system model can be arrived at by applying Eq.(7) to Eq.(6) and finally obtained as follows

$$\ddot{Y}=BU_c=\frac{1}{m}v \tag{8}$$

### 3. Adaptive Sliding mode Controller Design

The error output vector is defined as  $E=Y-Y_d$ , where  $Y_d$  is the desired position and  $Y$  is the current position. Then, Eq.(8) can be rewritten as

$$\ddot{E}=\frac{1}{m}v-\ddot{Y}_d+W \tag{9}$$

where  $W$  denotes the system's uncertainty and is assumed bounded  $\|W\| \leq P$ . Here, we assume that  $P$  is unknown.

In order to design a controller which possesses a better ability to gain high robustness and self-tuning property, two advanced control methods have been intergrated. In this paper, we will introduce the controller design and provide the stability analysis.

Based on the model Eq.(9), which is compactly re-expressed as

$$\begin{aligned} \dot{z}_1 &= \dot{E} \\ \dot{z}_2 &= \ddot{E} \\ \dot{z}_3 &= \ddot{E} = BU_c - \ddot{Y}_d + W \end{aligned} \tag{10}$$

Furthermore, we define the sliding surface variable  $S$  as

$$S=z_3+\zeta_1z_2+\zeta_2z_1 \tag{11}$$

Where  $\zeta_1, \zeta_2$  are positive constants. In this paper, we try to regulate the state  $Z=[z_1 \ z_2 \ z_3]^T$  to zero, in other words, the current outputs will achieve the goal of precise tracking of positioning and attitude. In the context of SMC, asymptotical convergence of the variable  $S$  to zero will apparently imply asymptotical convergence of  $Z$  as well to zero. To validate this, we

will need to investigate the dynamics of the sliding surface variable  $S$  as follows

$$\dot{S}=\dot{z}_3+\zeta_1z_2+\zeta_2z_1=BU_c-\ddot{Y}_d+W+\zeta_1z_2+\zeta_2z_1 \tag{12}$$

In addition to the SMC, an adaptive controller is applied for estimating the parameters of the system online while simultaneously controlling the system. After we have the estimates of the system parameters, these estimates can be adopted to the control command in Eq.(12) to form appropriate SMC with boundary layer as

$$U_c=B^{-1}[-k_1S-\hat{P}\text{sgn}(S)+\ddot{Y}_d-\zeta_1z_2-\zeta_2z_1] \tag{13}$$

where  $k_1$  is a positive constant,  $\hat{P}$  is the estimate of  $P$ , and  $\text{sgn}(\cdot)$  is the symbolic function.

Thus, substituting Eq.(13) into Eq.(12), we obtain

$$\begin{aligned} \dot{S} &= (\zeta_1z_2+\zeta_2z_1-\ddot{Y}_d+W)+B \\ & [B^{-1}(-k_1S-\hat{P}\text{sgn}(S)+\ddot{Y}_d-\zeta_1z_2-\zeta_2z_1)] \\ & = -k_1S-\hat{P}\text{sgn}(S)+W \end{aligned} \tag{14}$$

By appropriate gains  $k_1, \hat{P}, \zeta_1$  and  $\zeta_2$ , we can ensure the convergence of  $S$ . Hence, the state  $Z$  converges to zero, which means the tracking error  $E$  to zero, and the estimation error  $\hat{P}$  converging to zero.

Stability analysis: we define a Lyapunov function candidate  $V$ , which is a positive definite function

$$V=\frac{1}{2}S^T S+\frac{1}{2}\rho^{-1}\tilde{P}^2 \tag{15}$$

where the estimation error is defined as  $\tilde{P}=P-\hat{P}$ ,  $\hat{P}$  is the estimates of  $P$ , and  $\rho$  is positive.

The time derivative of the Lyapunov candidate function  $V$  can be found to be

$$\begin{aligned} \dot{V} &= S^T \dot{S} + \frac{1}{\rho} \tilde{P} \dot{\tilde{P}} \\ &= S^T [-k_1S-\hat{P}\text{sgn}(S)+W] - \frac{1}{\rho} \tilde{P} \dot{\tilde{P}} \\ &\leq -S^T k_1 S - \hat{P} \|S\| + \|S\| P - \frac{1}{\rho} \tilde{P} \dot{\tilde{P}} \end{aligned} \tag{16}$$

Using the adaptive control theory to establish bounds of parameter estimates in the presence of modeling error terms, the adaptive laws is devised as

$$\dot{\tilde{P}} = -\dot{\hat{P}} = -\rho \|S\| \tag{17}$$

After substituting Eq.(17) into Eq.(16), we can get

$$\begin{aligned} \dot{V} &\leq -S^T k_1 S - \hat{P} \|S\| + \|S\| P - \tilde{P} \|S\| \\ &= -S^T k_1 S - \hat{P} \|S\| + \hat{P} \|S\| \\ &< -S^T k_1 S < 0 \end{aligned} \tag{18}$$

where  $k_1 > 0$ .

According to Lyapunov stability theory, the tracking error  $\|E(t)\|$  will converge to zero. In other words, it achieves the goal of precise tracking of positioning.

#### 4. Simulation Results

In this section, a number of typical simulation results are presented, including the transient and the steady-state responses in different situations. The simulation results are provided to demonstrate the performance of the developed magnetic levitation system with controller presented in section 3. Based on these results, we will make some conclusions which are important for the future work in this research.

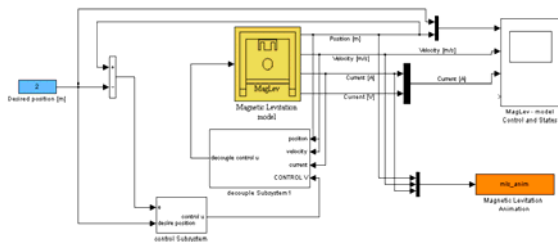


Fig.2. Block diagram of magnetic levitation system

In order to demonstrate the controlling performance of the magnetic levitation system more effectively, the block diagram of the system as shown in Fig.2. is constructed in this section. Base on the simulation block diagram, serials experiments are performed.

The tracking response performance of magnetic levitation vehicle system is shown in Fig.3., where the central levitation height can track the desire signal  $x(t)=0.5\sin t+1$ . From the position error curve shown in Fig.3., we can see that the error can be controlled within 0.03cm. The simulation result, as shown in Fig.4., demonstrates the ball can keep balance at the equilibrium point. The simulation results for repeating a 0.2cm step-train response in x-axis, as shown in Fig.5., indicates that the effective tracking performance of the proposed magnetic levitation vehicle system.

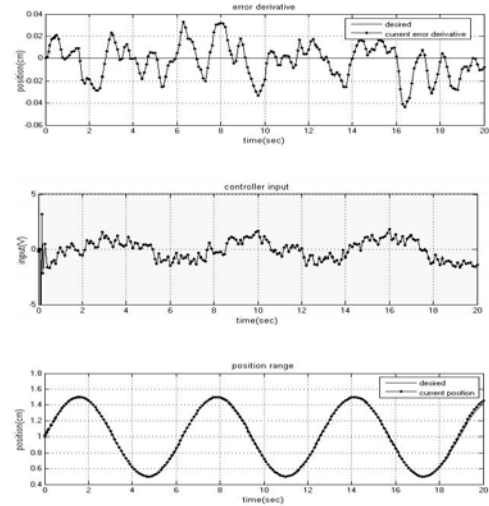
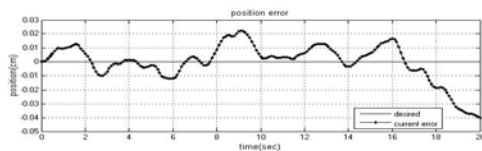


Fig.3. Tracking response performance of central levitation height along the x-axis with uncertainty bound

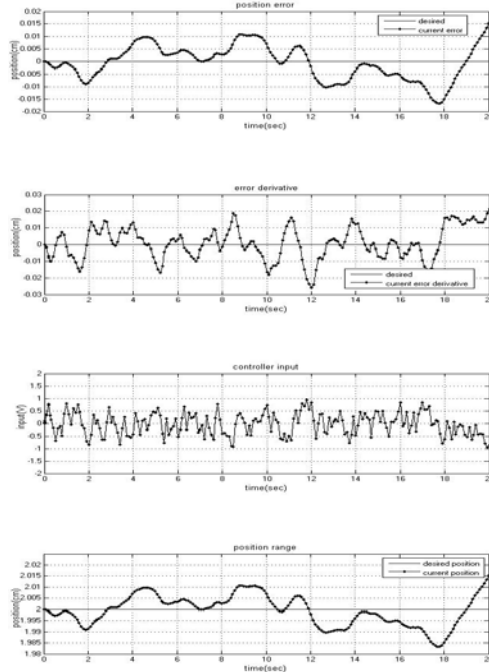


Fig.4. Results for central levitation height holding at 2cm

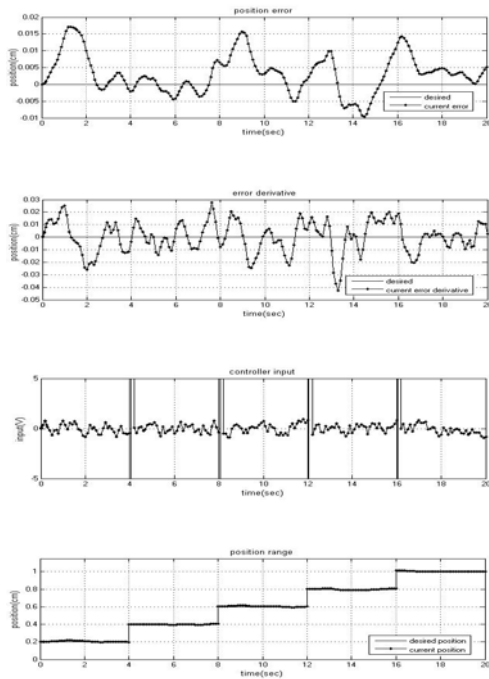


Fig.5.Results for repeating a 0.2cm step-train response

## 5. Conclusion

This paper does a research on the suspension control problem of the magnetic levitation vehicle system, which in nature, is served as strongly coupling, nonlinear and instability. The related analysis had been applied in to the maglev system. The linearization method and adaptive sliding mode controller can stable the system effectively.

## Acknowledgments

This work was supported by the National Basic Research Program of China(973 Program: 2012CB821 200,2012CB821201) and the NSFC(61134005, 609210 01,61327807).

## References

1. D. Chao, Y. Kato, and D. Spilman, A Sliding Mode and Classical Control of Magnetic Levitation Systems, *IEEE Control Syst. Mag.*, (1993)42-48.
2. B. V. Jayawant, P. K. Sinha, and D. G. Aylwin, Feedback control systems for dc electromagnets in passenger-carrying vehicles. *International Journal of Control* 24(5) (1976) 627-639.
3. M. Proise, System Concept Definition of the Grumman Superconducting Electromagnetics Suspension (EMS)

Maglev Design, *Maglev 93 Conf. Argonne National Laboratory*, (1993) 19-21.

4. E. E. Covert, M. Vlajinac, T. Stephens, and M. Finston, Magnetic balance and suspension systems for use with wind tunnels, *Progress in Aerospace Sciences* (14) (1973)27-107.
5. M.Y. Chen, M.J. Wang, and L.C. Fu, Modeling and controller design of a maglev guiding system for application in precision positioning. *Industrial Electronics, IEEE Transactions on* 50(3) (2003)493-506.
6. H. Chen, Z. Long, W. Chang, Fault tolerant control research for high-speed maglev system with sensor failure, *Intelligent Control and Automation, WCICA*. (The Sixth World Congress on. IEEE, 2006), pp. 2281-2285.