Journal of Robotics, Networking and Artificial Life, Vol. 5, No. 2 (September 2018) 118-121

Tracking/Robust Trade-off Design of a Sampled-data PID Controller for Second-order Plus Dead-time Systems

Ryo Kurokawa and Takao Sato
Department of Mechanical Engineering, University of Hyogo
2167 Shosha, Himeji, Hyogo 671-2280 Japan
E-mail: tsato@eng.u-hyogo.ac.jp

Ramon Vilanova
Department of Telecommunications and Systems Engineering, Universitat Autònomade Barcelona
Edifici Q-Campus de la UAB, 08193 Bellaterra, Barcelona, Spain
E-mail: ramon.vilanova@uab.cat

Yasuo Konishi
Department of Mechanical Engineering, University of Hyogo
2167 Shosha, Himeji, Hyogo 671-2280 Japan
E-mail: konishi@eng.u-hyogo.ac.jp

Abstract

In this paper, we propose a new design method of a second-order plus dead-time (SOPDT) sampled-data Proportional-Integral-Derivative (PID) control system, where the continuous-time plant is controlled using the discrete-time controller. The proposed control system is designed so that the tracking performance is optimized subject to the stability margin constraint. In the present study, the servo and regulation optimal controllers are designed. Finally, the effectiveness of the proposed method is demonstrated through numerical examples.

Keywords: PID control, Sampled-data system, SOPDT system, Sensitivity function, Robust

1. Introduction

Proportional-Integral-Derivative (PID)[1,2] control has been widely used in industry. Since the performance of PID control depends on the tuning parameters, additional tuning methods have been studied recently. Although the stability of a control system is critical, its tracking performance is also important. However, because of the trade-off relationship between stability and tracking performance, they cannot be optimized simultaneously. Arrieta and Vilanova[3,4] proposed a simple PID tuning method that optimizes the tracking performance subject to a prescribed robust stability. In this method, the optimal PID parameters are decided based on a first-order plus dead-time (FOPDT) continuous-time system. In order to design a discrete-time control system, Tajika et al.[5] proposed a design method for controlling a discrete-time FOPDT system. The present study discusses a design method of the PID controller for controlling a second-order plus dead-time (SOPDT) system, in which the continuous-time plant is controlled using the discrete-time controller. In the proposed method, both servo and regulation optimized control methods are designed.

Finally, the effectiveness of the proposed method is demonstrated through numerical examples.

2. Description of the Control System

Consider the continuous-time controlled plant given as follows:

$$P(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} e^{-Ls} \quad (1)$$

where K is the plant gain, ω_n is the natural angular frequency, ζ is the damping coefficient, and L is the dead-time. In the present study, we discuss the design method of the sampled-data control system using the following discrete-time PID control law:

$$u(k) = C_e(z^{-1})e(k) + C_y(z^{-1})y(k) \quad (2)$$

$$C_d(z^{-1}) = C_e(z^{-1}) + C_y(z^{-1})$$

$$C_e(z^{-1}) = K_p\left\{1 + \frac{T_s}{T_i(1 - z^{-1})}\right\}$$

$$C_y(z^{-1}) = K_p\left\{\frac{T_d(1 - z^{-1})}{T_s}\right\}$$

where $u(k)$ is the control input, $y(k)$ is the plant output, $e(k)(= r(k) - y(k))$ is the control error, and $r(k)$ is the reference. Moreover, T_s, K_p, T_i, and T_d are the sampling time, the proportional gain, the integral time, and the differential time, respectively.

3. Definition of the Optimization Problem

As the constraint condition, the stability margin is defined using the sensitivity function, and the evaluation function for the tracking performance is also defined.

3.1. *Constraint condition*

The sensitivity function $S_f(z^{-1})$ is defined as follows:

$$S_f(z^{-1}) = \frac{1}{1 + C_d(z^{-1})P_d(z^{-1})} \quad (3)$$

where $P_d(z^{-1})$ is the discrete-time controlled plant. Using the sensitivity function, the constraint condition is defined as follows:

$$|M_s - M_s^d| = 0 \quad (4)$$

$$M_s = \max_{\omega}\left|S_f(e^{-j\omega})\right|$$

where M_s is the maximum value of the sensitivity function, and M_s^d is the desired value selected by the designer. The recommended range of M_s^d is from 1.4 to 2.0[1]. The smaller the value of M_s, the larger the stability margin. On the other hand, the larger the value of M_s, the better the tracking performance, although the stability margin becomes small.

3.2. *Evaluation function*

In the present study, the evaluation function J is defined as the integral absolute error:

$$J = \sum_{k=0}^{\infty}|e(k)| = \sum_{k=0}^{\infty}|r(k) - y(k)| \quad (5)$$

A trade-off relationship exists between the servo performance and the regulation performance. In the present study, the PID parameters are optimized for the servo and regulation control, respectively.

4. Controller Design

The PID parameters are optimized for a normalized system, and hence, dimensionless parameters are defined as $\tau = L\omega_n$, $h = T_s\omega_n$, $\kappa_p = K_pK$, $\tau_i = T_i\omega_n$, and $\tau_d = T_d\omega_n$. The range of these parameters are set as $0.1 \le \tau \le 1.0$, $0.01 \le h \le 0.10$, and $0.3 \le \zeta \le 1.2$.
In the proposed method, the constrained optimal problem is preliminarily solved for a designated finite plant, which is defined by discrete τ, h, and ζ, and the data set in which the optimal normalized PID parameters for discrete τ, h, and ζ, is obtained. In Fig. 1, the obtained normalized PID parameters are plotted by ◦, where $M_s^d = 1.4$ and $T_s = 0.01$.

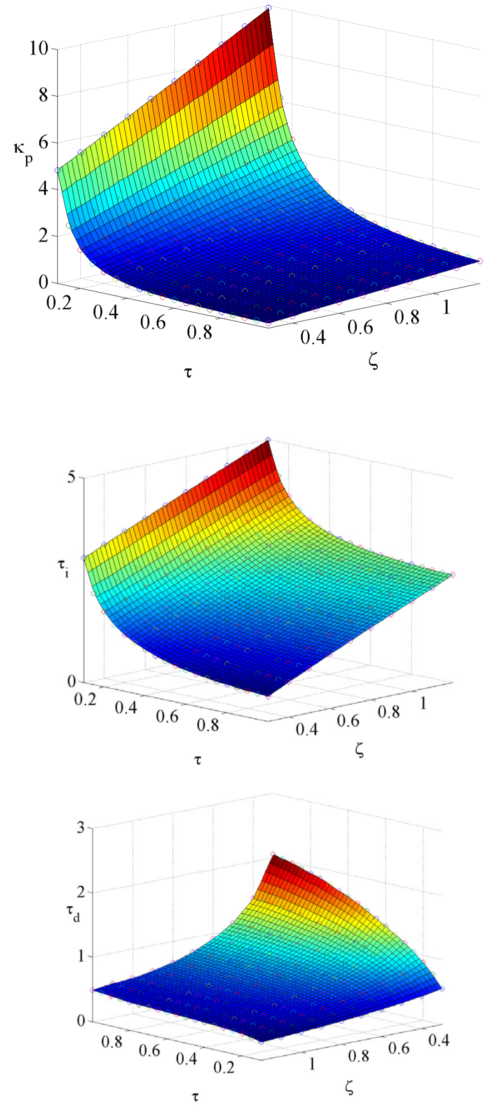

Fig. 1. Relationships among τ, ζ, and κ_p, τ_i and τ_d (servo design, $M_S^d = 1.4$ and $T_s = 0.01$)

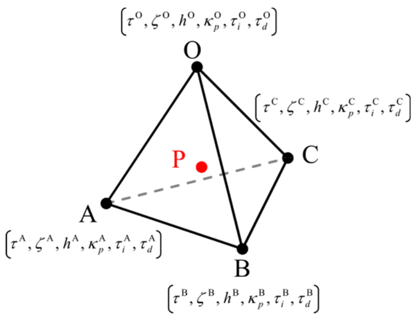

Fig. 2. Image of the linear interpolation

The desired normalized PID parameters for an arbitrary plant are decided by the linear interpolation from the data set. Practically speaking, the interpolated parameters are calculated using the nearest four points, as shown in Fig. 2. From this figure, the vector equation is obtained as follows:

$$\overrightarrow{\mathrm{OP}} = \alpha\overrightarrow{\mathrm{OA}} + \beta\overrightarrow{\mathrm{OB}} + \gamma\overrightarrow{\mathrm{OC}} \quad (6)$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$0 \le \gamma \le 1$$

where point O $[\tau^{\mathrm{O}}, \zeta^{\mathrm{O}}, h^{\mathrm{O}}, \kappa_p^{\mathrm{O}}, \tau_i^{\mathrm{O}}, \tau_d^{\mathrm{O}}]$, A $[\tau^{\mathrm{A}}, \zeta^{\mathrm{A}}, h^{\mathrm{A}}, \kappa_p^{\mathrm{A}}, \tau_i^{\mathrm{A}}, \tau_d^{\mathrm{A}}]$, B $[\tau^{\mathrm{B}}, \zeta^{\mathrm{B}}, h^{\mathrm{B}}, \kappa_p^{\mathrm{B}}, \tau_i^{\mathrm{B}}, \tau_d^{\mathrm{B}}]$, and C $[\tau^{\mathrm{C}}, \zeta^{\mathrm{C}}, h^{\mathrm{C}}, \kappa_p^{\mathrm{C}}, \tau_i^{\mathrm{C}}, \tau_d^{\mathrm{C}}]$ are the nearest points of the desired $[\tau^{\mathrm{P}}, \zeta^{\mathrm{P}}, h^{\mathrm{P}}]$. Then, Eq. (6) is rearranged as follows:

$$\kappa_p^{\mathrm{P}} = \kappa_p^{\mathrm{O}} + \alpha(\kappa_p^{\mathrm{A}} - \kappa_p^{\mathrm{O}}) + \beta(\kappa_p^{\mathrm{B}} - \kappa_p^{\mathrm{O}}) + \gamma(\kappa_p^{\mathrm{C}} - \kappa_p^{\mathrm{O}})$$
$$\tau_i^{\mathrm{P}} = \tau_i^{\mathrm{O}} + \alpha(\tau_i^{\mathrm{A}} - \tau_i^{\mathrm{O}}) + \beta(\tau_i^{\mathrm{B}} - \tau_i^{\mathrm{O}}) + \gamma(\tau_i^{\mathrm{C}} - \tau_i^{\mathrm{O}})$$
$$\tau_d^{\mathrm{P}} = \tau_d^{\mathrm{O}} + \alpha(\tau_d^{\mathrm{A}} - \tau_d^{\mathrm{O}}) + \beta(\tau_d^{\mathrm{B}} - \tau_d^{\mathrm{O}}) + \gamma(\tau_d^{\mathrm{C}} - \tau_d^{\mathrm{O}})$$

Solving these equations, the desired κ_p^{P}, τ_i^{P}, and τ_d^{P} for $[\tau^{\mathrm{P}}, \zeta^{\mathrm{P}}, h^{\mathrm{P}}]$ are obtained, where α, β, and γ are decided based on the following equations:

$$\tau^{\mathrm{P}} - \tau^{\mathrm{O}} = \alpha(\tau^{\mathrm{A}} - \tau^{\mathrm{O}}) + \beta(\tau^{\mathrm{B}} - \tau^{\mathrm{O}}) + \gamma(\tau^{\mathrm{C}} - \tau^{\mathrm{O}})$$
$$\zeta^{\mathrm{P}} - \zeta^{\mathrm{O}} = \alpha(\zeta^{\mathrm{A}} - \zeta^{\mathrm{O}}) + \beta(\zeta^{\mathrm{B}} - \zeta^{\mathrm{O}}) + \gamma(\zeta^{\mathrm{C}} - \zeta^{\mathrm{O}})$$
$$h^{\mathrm{P}} - h^{\mathrm{O}} = \alpha(h^{\mathrm{A}} - h^{\mathrm{O}}) + \beta(h^{\mathrm{B}} - h^{\mathrm{O}}) + \gamma(h^{\mathrm{C}} - h^{\mathrm{O}})$$

In Fig. 1, the interpolated parameters are plotted over the discrete calculated optimal parameters. Furthermore, M_s is calculated for both the preliminarily solved and interpolated systems using the approximation method, and the obtained M_s values are shown in Table 1. This result reveals that the proposed decision method is sufficiently effective.

Table 1. Obtained M_s

	Servo design			**Regulation design**		
M_s^d	Min	Mean	Max	Min	Mean	Max
1.4	1.398	1.403	1.440	1.399	1.403	1.453
1.6	1.599	1.605	1.668	1.597	1.605	1.663
1.8	1.790	1.807	1.909	1.798	1.807	1.897
2.0	1.996	2.010	2.156	1.997	2.009	2.137

Journal of Robotics, Networking and Artificial Life, Vol. 5, No. 2 (September 2018) 118-121

5. Numerical Simulation

In this section, the effectiveness of the proposed method is confirmed.

5.1. *Control performance for various values of* ζ

First, the control performance is confirmed for ζ. The controlled plant is defined as $K = 4.2$, $\omega_n = 1.13$, and $L = 0.44$ in Eq. (1), and $T_s = 0.018$. Here, we consider four pattern damping coefficients: $\zeta^1 = 0.451$, $\zeta^2 = 0.69$, $\zeta^3 = 1.0$, and $\zeta^4 = 1.199$. The control results are shown in Fig. 3. The reference value is set to 1.0, and the unit step disturbance signal is added after 20 s. Figure 3 shows that the proposed method is effective for under- and over-damping systems.

5.2. *Verification of stability margin*

Next, the stability margin is confirmed. Here, the controlled plant is defined as $K = 2.02$, $\omega_n = 0.91$, $\zeta = 0.33$, and $L = 0.98$ in Eq. (1), and $T_s = 0.05$. After 40 s, the dynamics is changed to $K = 2.6$, $\omega_n = 1.3$, $\zeta = 0.43$, and $L = 0.43$ as the model variation. Furthermore, M_s^d is varied as 1.4, 1.6, 1.8, and 2.0, respectively, and the control results are compared. The obtained results are shown in Fig. 4. The reference value is 1.0, and the unit step disturbance signal is added after 20 s. The model variation is caused at 40 s. Figure 3 shows that the smaller the value of M_s^d, the larger the stability margin, and vice versa. On the other hand, the larger the value of M_s^d, the better the tracking performance, and vice versa.

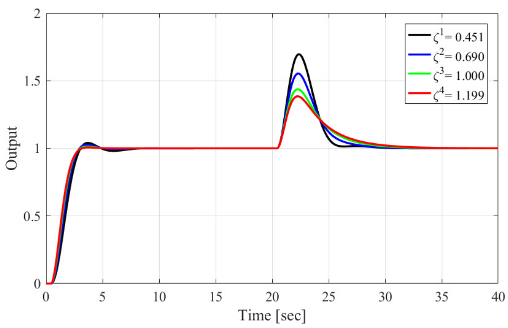

Fig. 3. Output responses for each damping coefficient ζ^i (servo design and $M_s^d = 1.4$)

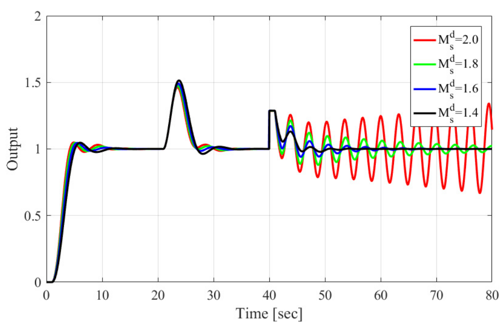

Fig. 4. Output responses for each M_s^d

Conclusion

In the present study, we have proposed a new design method for controlling an SOPDT sampled-data system, where the continuous-time plant is controlled by the discrete-time PID control law. In the proposed method, the PID parameters are designed for the normalized system, and the tracking performance is optimized subject to the assignedM_s^d. Finally, the effectiveness of the proposed method is demonstrated through numerical examples.

Acknowledgements

The present study was supported by JSPS KAKENHI Grant Number JP16K06425. The authors would like to express their sincere thanks for the support from the Japan Society for the Promotion of Science.

References

1. K. J. Åström and T. Hägglund, *Advanced PID Control*, ISA-Instrumentation, Systems, and Automation Society, 2006.
2. R. Vilanova and A. Visioli, *PID Control in the Third Millennium* (Springer 2012, UK).
3. O. Arrieta and R. Vilanova, Simple PID Tuning Rules with Guaranteed M_s Robustness Achievement, *IFAC Proceedings* **44**(1), 2011, pp. 12042-1204.
4. O. Arrieta and R. Vilanova, Simple Servo/Regulation Proportional-Integral-Derivative (PID) Tuning Rules for Arbitrary M_s-Based Robustness Achievement, *Industrial & Engineering Chemistry Research* **51**(6), 2012, pp. 2666-2674.
5. H. Tajika, T. Sato, R. Vilanova and Y. Konishi, Optimal PID Control in Discrete Time Using a Sensitivity Function, *23th Mediterranean Conference on Control and Automation (MED)*, 2015, pp. 249-254.