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Research Article

Analysis and Stability Control of a Novel 5D Hyperchaotic System

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ABSTRACT

In this paper, a novel Five-Dimensional (5D) autonomous hyperchaotic system is presented, and the characteristics of the 5D system are given in brief. For stability control of the 5D hyperchaotic system, a linear feedback controller and its simplification are designed via the Lyapunov stability theory, so that the 5D system is no longer hyperchaotic but globally asymptotically converges to the equilibrium point at the origin. The numerical simulation results are given to illustrate the feasibility and effectiveness of the method.

1. INTRODUCTION

Hyperchaos was first proposed by Otto Rössler in 1979 [1]. Since then, many novel hyperchaotic systems have been formulated. To obtain hyperchaos, the system need to satisfy the following two important requisites. Firstly, the minimal dimension of the phase space that embeds a hyperchaotic attractor should be at least four, which requires the minimum number of coupled first-order autonomous ordinary differential equations to be four. Secondly, the number of terms in the coupled equations giving rise to instability should be at least two, of which at least one should have a nonlinear function [2]. Therefore, hyperchaos is much more complicated than chaos, and it has greater engineering significance and application prospect in signal processing, secure communication and so on.

In this paper, a novel 5D hyperchaotic system, which has been introduced in Wei and Niu [3], is reviewed. Stability control of the 5D system would be discussed, and some simulation results would be given to demonstrate the validity of the designed linear feedback controllers.

2. THE NOVEL 5D HYPERCHAOTIC SYSTEM

The dynamic equations of the novel 5D hyperchaotic system are formulated as

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= (c-a)x + cy + w - xz, \\ \dot{z} &= -bz + xy, \\ \dot{v} &= mw, \\ \dot{w} &= -y - hv, \end{aligned} \tag{1}$$

where $x, y, z, v, w \in R$ are state variables, and $a = 23$, $b = 3$, $c = 18$, $m = 12$ and $h = 4$ [3].

Let the initial values of the 5D system (1) be $(x_0, y_0, z_0, v_0, w_0) = (1, 1, 1, 1, 1)$, then the Lyapunov exponents respectively are $\lambda_1 = 0.8732 > 0$, $\lambda_2 = 0.1282 > 0$, $\lambda_3 = -0.0013 \approx 0$, $\lambda_4 = -0.5770 < 0$ and $\lambda_5 = -8.4231 < 0$. It indicates that the 5D system (1) is hyperchaotic. The attractors of the 5D hyperchaotic system (1) are shown in Figure 1.

3. HYPERCHAOS CONTROL OF THE 5D SYSTEM

3.1. Formulation of the Controlled System

The controlled system is represented as

$$\begin{aligned} \dot{x} &= a(y-x) + u_{c1}, \\ \dot{y} &= (c-a)x + cy + w - xz + u_{c2}, \\ \dot{z} &= -bz + xy + u_{c3}, \\ \dot{v} &= mw + u_{c4}, \\ \dot{w} &= -y - hv + u_{c5}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \boldsymbol{u}_c &= \begin{bmatrix} u_{c1} & u_{c2} & u_{c3} & u_{c4} & u_{c5} \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} -k_1x & -k_2y & -k_3z & -k_4v & -k_5w \end{bmatrix}^{\mathrm{T}}, \end{aligned}$$

and $k_1, k_2, k_3, k_4, k_5 \geq 0$.

3.2. Design of the Linear Feedback Controller

Theorem 1. *Let $\boldsymbol{x} = \boldsymbol{0}$ be an equilibrium point for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{f}: D \to R^n$ is a locally Lipschitz map from a domain $D \subset R^n$ into R^n. Let $V: R^n \to R$ be a continuously differentiable function such that*

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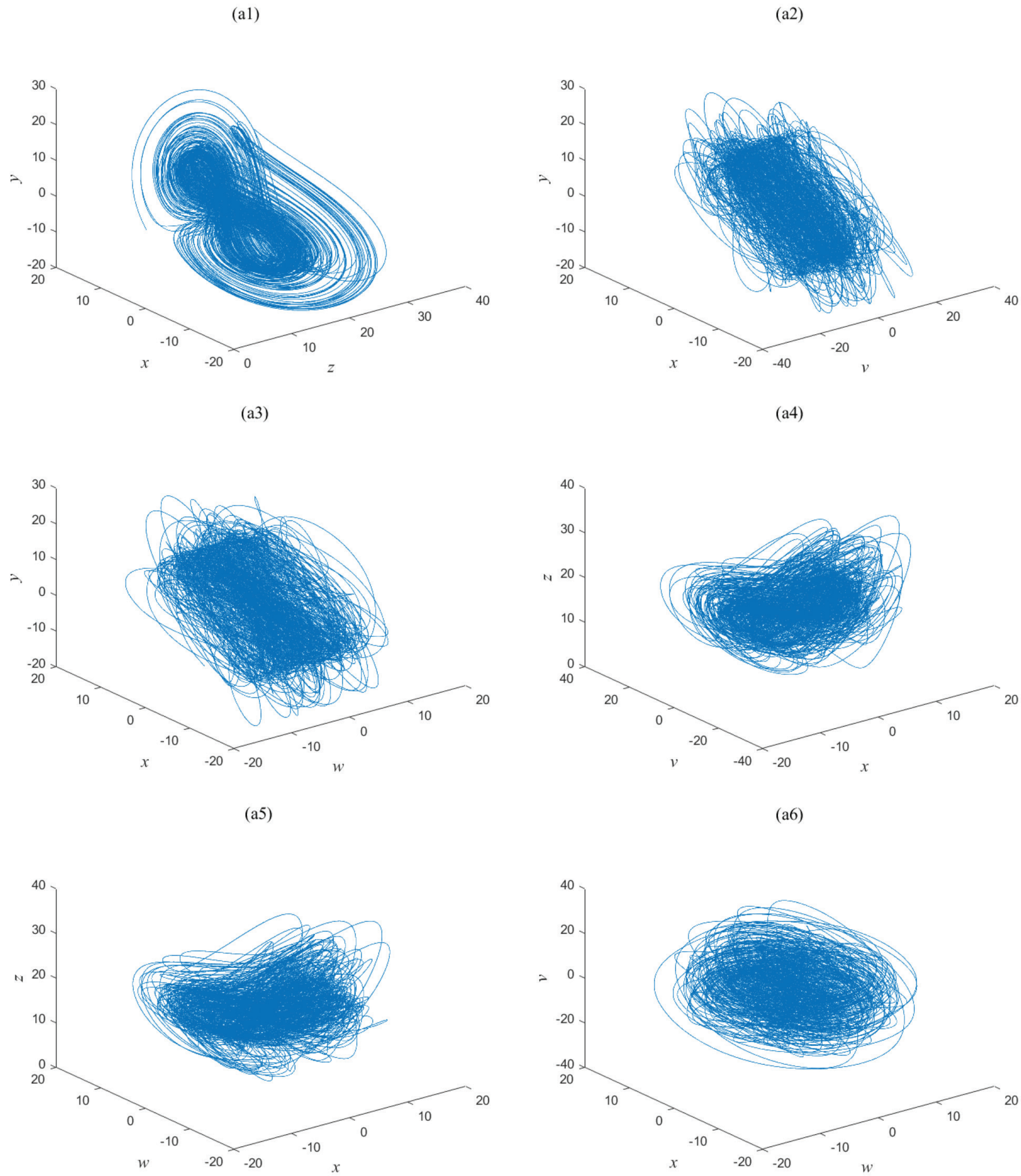

Figure 1 | Attractors of the 5D hyperchaotic system: (a1) *z-x-y*; (a2) *v-x-y*; (a3) *w-x-y*; (a4) *x-v-z*; (a5) *x-w-z*; (a6) *w-x-v*.

$$V(\mathbf{0}) = 0 \quad \text{and} \quad V(\boldsymbol{x}) > 0, \quad \forall \boldsymbol{x} \neq \mathbf{0}$$
$$\|\boldsymbol{x}\| \to \infty \Rightarrow V(\boldsymbol{x}) \to \infty$$
$$\dot{V}(\boldsymbol{x}) < 0, \quad \forall \boldsymbol{x} \neq \mathbf{0}$$

then $\boldsymbol{x} = \mathbf{0}$ *is globally asymptotically stable* [4].

From Theorem 1, take a continuously differentiable function

$$V = \frac{1}{2}\left(x^2 + y^2 + z^2 + \frac{h}{m}v^2 + w^2\right) \tag{3}$$

as a Lyapunov function candidate for the controlled system (2). Then, the derivative $\dot{V}$ is given by

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + \frac{h}{m}v\dot{v} + w\dot{w} \\ &= -(k_1 + a)x^2 + cxy - (k_2 - c)y^2 \\ &\quad -(k_3 + b)z^2 - k_4\frac{h}{m}v^2 - k_5w^2 \\ &\leq -\left(k_1 + a - \frac{c}{2}\right)x^2 - \left(k_2 - \frac{3}{2}c\right)y^2 \\ &\quad -(k_3 + b)z^2 - k_4\frac{h}{m}v^2 - k_5w^2. \end{aligned} \tag{4}$$

For $\dot{V} < 0$, the parameters k_1, k_2, k_3, k_4 and k_5 should satisfy that

$$\begin{array}{c} k_1 + a - \frac{c}{2} > 0, \\ k_2 - \frac{3}{2}c > 0, \\ k_3 + b > 0, \\ k_4 \frac{h}{m} > 0, \\ k_5 > 0, \end{array} \Rightarrow \begin{array}{l} k_1 > \frac{c}{2} - a, \\ k_2 > \frac{3}{2}c, \\ k_3 > -b, \\ k_4 > 0, \\ k_5 > 0, \end{array} \Rightarrow \begin{array}{l} k_1 = 0, \\ k_2 = 30, \\ k_3 = 0, \\ k_4 = 1, \\ k_5 = 1. \end{array}$$

From Theorem 1, the controlled system (2) is globally asymptotically stable at the origin. Thus, the controller $\boldsymbol{u}_c$ can be designed as

$$\begin{aligned} \boldsymbol{u}_c &= \begin{bmatrix} u_{c1} & u_{c2} & u_{c3} & u_{c4} & u_{c5} \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} 0 & -30y & 0 & -v & -w \end{bmatrix}^{\mathrm{T}}. \end{aligned} \tag{5}$$

3.3. Numerical Simulation under the Controller u_c

The curves of the state variables of the controlled system (2) before and after adding the controller $\boldsymbol{u}_c$ are shown in Figures 2 and 3 respectively. Comparing Figure 3 with Figure 2, it can be found that

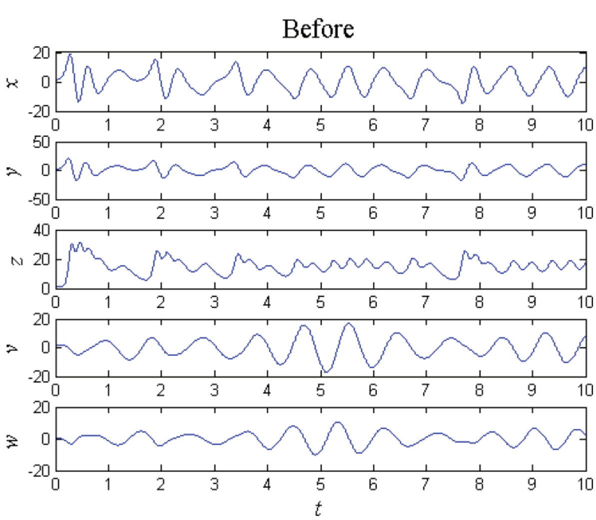

Figure 2 | Curves of the state variables before adding $\boldsymbol{u}_c$.

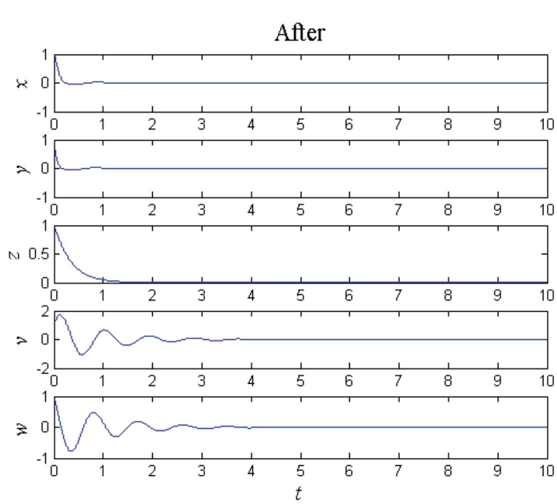

Figure 3 | Curves of the state variables after adding $\boldsymbol{u}_c$.

the state variables x, y, z, v and w converge to zero asymptotically and rapidly. Meanwhile, the Lyapunov exponents of the controlled system (2) are $\lambda c_1 = -1.0321$, $\lambda c_2 = -1.0327$, $\lambda c_3 = -3.0000$, $\lambda c_4 = -17.4666$ and $\lambda c_5 = -17.4687$, which are all negative. It implies that the controlled system (2) is no longer hyperchaotic but asymptotically stable at the origin. It illustrates that the linear feedback controller $\boldsymbol{u}_c$ is feasible and effective for hyperchaos control of the 5D system (2).

3.4. Simplification of the Controller u_c

Corollary 1. *Let $\boldsymbol{x} = \boldsymbol{0}$ be an equilibrium point for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{f}: D \to R^n$ is a locally Lipschitz map from a domain $D \subset R^n$ into R^n. Let $V: R^n \to R$ be a continuously differentiable, radially unbounded, positive definite function such that $\dot{V}(\boldsymbol{x}) \le 0$ for all $\boldsymbol{x} \in R^n$. Let $S = \{\boldsymbol{x} \in R^n | \dot{V}(\boldsymbol{x}) = 0\}$ and suppose that no solution can stay identically in S, other than the trivial solution $\boldsymbol{x}(t) \equiv \boldsymbol{0}$. Then, the origin is globally asymptotically stable* [4].

Assume that the minimum number of the feedback variables might be equal to the number of the positive Lyapunov exponents [5]. The 5D hyperchaotic system (1) has two positive Lyapunov exponents, but there are three feedback variables in Equation (5). Still take Equation (3) as a Lyapunov function candidate for the controlled system (2). Now let $k_4 = 0$ and substitute $k_1 = k_3 = k_4 = 0$ into Equation (4). Then, the derivative $\dot{V}$ is reduced to

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + \frac{h}{m}v\dot{v} + w\dot{w} \\ &= -ax^2 + cxy - (k_2 - c)y^2 - bz^2 - k_5w^2 \\ &\le -\left(a - \frac{c}{2}\right)x^2 - \left(k_2 - \frac{3}{2}c\right)y^2 - bz^2 - k_5w^2. \end{aligned} \tag{6}$$

For $\dot{V} \le 0$, the parameters k_2 and k_5 should satisfy that

$$\begin{array}{c} k_2 - \frac{3}{2}c > 0, \\ k_5 > 0, \end{array} \Rightarrow \begin{array}{l} k_2 > \frac{3}{2}c, \\ k_5 > 0, \end{array} \Rightarrow \begin{array}{l} k_2 = 30, \\ k_5 = 1. \end{array}$$

From Corollary 1, to find $S = \{\boldsymbol{x} \in R^5 | \dot{V}(\boldsymbol{x}) = 0\}$, note that

$$\dot{V} = 0 \Rightarrow x = y = z = w = 0$$

Hence, $S = \{\boldsymbol{x} \in R^5 | x = y = z = w = 0\}$.

Let $\boldsymbol{x}(t)$ be a solution that belongs identically to $S = \{\boldsymbol{x} \in R^5 | x = y = z = w = 0\}$, so that

$$\begin{aligned} & x(t) = y(t) = z(t) = w(t) \equiv 0 \\ \Rightarrow\ & \dot{x}(t) = \dot{y}(t) = \dot{z}(t) = \dot{v}(t) = \dot{w}(t) \equiv 0 \\ \Rightarrow\ & v(t) \equiv 0 \end{aligned}$$

Therefore, the only solution that can stay identically in $S = \{\boldsymbol{x} \in R^5 | \dot{V}(\boldsymbol{x}) = 0\}$ is the trivial solution $\boldsymbol{x}(t) \equiv \boldsymbol{0}$. Thus, the origin is globally asymptotically stable. Finally, the controller $\boldsymbol{u}_c$ in Equation (5) is simplified as

$$\begin{aligned} \boldsymbol{u}_{cs} &= \begin{bmatrix} u_{cs1} & u_{cs2} & u_{cs3} & u_{cs4} & u_{cs5} \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} 0 & -30y & 0 & 0 & -w \end{bmatrix}^{\mathrm{T}}. \end{aligned} \tag{7}$$

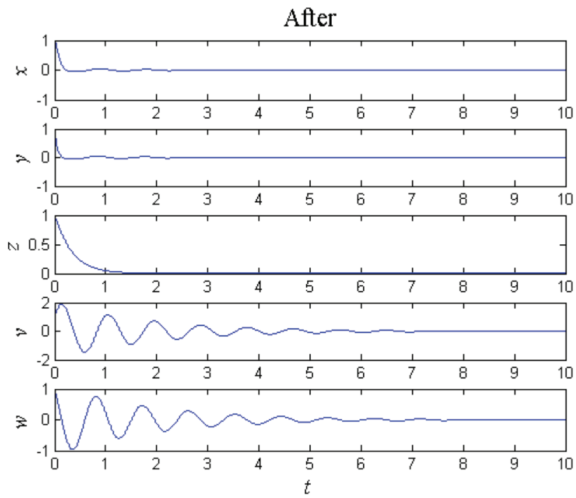

Figure 4 | Curves of the state variables after adding $\boldsymbol{u}_{cs}$.

3.5. Numerical Simulation under the Simplified Controller u_{cs}

Let the initial values still be $(x_0, y_0, z_0, v_0, w_0) = (1, 1, 1, 1, 1)$, then the curves of the state variables of the controlled system (2) before and after adding the simplified controller $\boldsymbol{u}_{cs}$ are shown in Figures 2 and 4 respectively. Comparing Figure 4 with Figure 2, it can be found that the state variables x, y, z, v and w converge to zero asymptotically and rapidly. Meanwhile, the Lyapunov exponents of the controlled system (2) are $\lambda cs_1 = -0.53246$, $\lambda cs_2 = -0.53248$, $\lambda cs_3 = -3.0000$, $\lambda cs_4 = -17.4664$ and $\lambda cs_5 = -17.4687$, which are all negative. It implies that the controlled system (2) is no longer hyperchaotic but asymptotically stable at the origin. It illustrates that the simplified linear feedback controller $\boldsymbol{u}_{cs}$ is also feasible and effective for hyperchaos control of the 5D system (2). Furthermore, the simplified controller $\boldsymbol{u}_{cs}$ only has two feedback variables, such that it is easier to implement via circuit than the controller $\boldsymbol{u}_c$.

4. CONCLUSION

In this paper, a novel 5D hyperchaotic system is reviewed. For hyperchaos control of the 5D system, a linear feedback controller and its simplification are designed via the Lyapunov stability theory. The numerical simulation results demonstrate the validity of the controllers. The study in this paper has some engineering significance.

CONFLICTS OF INTEREST

The author declares no conflicts of interest.

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AUTHOR INTRODUCTION

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She received the PhD degree in control science and engineering from Tianjin University, China, in 2014. She is currently a lecturer with the College of Electronic Information and Automation, Tianjin University of Science & Technology, China. Her main research interests are analysis, control, synchronization and circuit implementation of chaotic and hyperchaotic systems.