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Research Article

Tracking Control of Mobile Robots based on Rhombic Input Constraints

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ABSTRACT

This paper focuses on the trajectory tracking control algorithm for Differential Wheeled Mobile Robots (DWMRs) based on rhombic input constraints. The kinematics and dynamics model of DWMRs are established, and vector analysis method is used to design the controller when the linear velocity and angular velocity of DWMRs were not mutually independently. Through the simulation of tracking 8-shaped curve, a good control performance is obtained.

1. INTRODUCTION

Differential Wheeled Mobile Robots (DWMRs) are widely used in today's society. There are many methods have been used in controller design for trajectory tracking. Sliding mode control [1], backstepping control [2], robust control [3], fuzzy control [4], active disturbance rejection control [5] etc. are used to solve tracking control problem. From a practical perspective, the input constraints must be considered when designing controller, but the current situation is that most of the research does not consider the mutual constraints relationship between the linear velocity v and the angular velocity ω of the mobile robot, they usually assume that the input constraints of the robots' linear velocity v and angular velocity ω are mutually independently, that is, $|v| \le m_1$, $|\omega| \le m_2$, where m_1 and m_2 are positive constants. The real situation is that input field of DWMRs is the rhombic area defined by $|v/m| + |\omega l/m| \le 1$ as shown in Figure 1, and m represents the maximum velocity of two drive wheels, l represents half of the distance between the two drive wheels, the proof process will be given later. If a differential wheeled mobile robot uses the controller designed in Su and Zheng [6], the rectangular area where v and ω are independently needs to be obtained from the rhombic area mentioned above, it can be determined by $|v| \le m/2$ and $|\omega| \le m/2l$. So we can see that the actual area where v and ω are mutually independently is only half of the hypothetical rectangular area. Also the mobility of DWMRs cannot be fully utilized. Rhombic input constraints are considered first time in Chen et al. [7], it proposed a geometric analysis method to design time-varying feedback parameters.

2. PROBLEM STATEMENT

2.1. Rhombic Input Constraints

As shown in Figure 2, v_l and v_r respectively represent the velocity of robot's driving wheels, and their maximum velocity is m, that is $v_l \le m$ and $v_r \le m$. Usually v and ω of DWMRs are used as control inputs, and their relationship with the velocity of the driving wheel is

$$v = \frac{(v_l + v_r)}{2} \tag{1}$$

$$\omega = \frac{(v_r - v_l)}{2l} \tag{2}$$

Thus v and ω are constrained by

$$\begin{cases} -(m+v)/l \le \omega \le (m+v)/l, v \in [-m, 0) \\ -(m-v)/l \le \omega \le (m-v)/l, v \in [0, m] \end{cases} \tag{3}$$

The above is collated into one expression:

Formula (3) can be sorted into one expression:

$$|v/m| + |\omega\ l/m| \le 1 \tag{4}$$

Formula (4) can be expressed as the solid black rhombus in Figure 1. So far, the independent rhombic area of v and ω is obtained.

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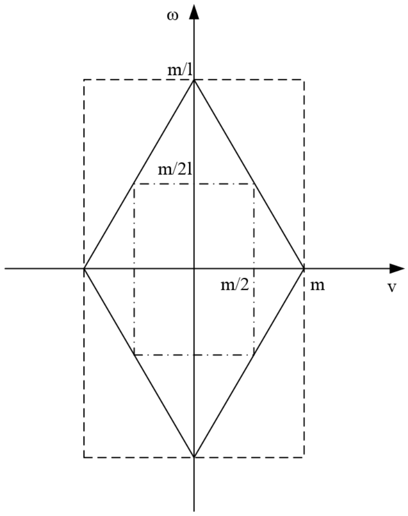

Figure 1 | Rectangular and diamond constraints.

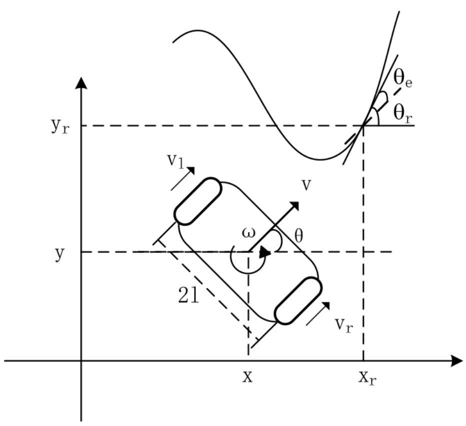

Figure 2 | Trajectory tracking of DWMRs.

2.2. Tracking Control Based on Rhombic Input Constraints

The kinematics and dynamics equations of two-wheel differential mobile robots is

$$\dot{x} = v\cos\theta,\quad \dot{y} = v\sin\theta,\quad \dot{\theta} = \omega \tag{5}$$

(x, y) is the center point coordinates of DWMRs and θ is used to indicate its azimuth angle (see Figure 2).

Assumption 1. The input constraint of DWMRs is Equation (4), and its reference trajectory satisfies:

$$\dot{x}_r = v_r\cos\theta_r,\quad \dot{y}_r = v_r\sin\theta_r,\quad \dot{\theta}_r = \omega_r \tag{6}$$

and

$$\left|v_r/m\right| + \left|\omega_r l/m\right| \le 1 - l\varepsilon\,|\,m \tag{7}$$

Among them, $(x_r, y_r, \theta_r, v_r, \omega_r)$ is the target values of $(x, y, \theta, v, \omega)$, where ε is a constant satisfies $0 < \varepsilon < m/l$.

Remark 1. We ensure the traceability of the trajectory by introducing a constant ε in formula (7).

Figure 2 shows that system errors of DWMRs are defined as:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{8}$$

The tracking errors system can be obtained by deriving the two sides of the above formula (8)

$$\begin{aligned} \dot{x}_e &= v_r\cos\theta_e - v + \omega y_e \\ \dot{y}_e &= v_r\sin\theta_e - \omega x_e \\ \dot{\theta}_e &= v_r\omega_r - \omega \end{aligned} \tag{9}$$

Now our task is how to design the controller with satisfying the input constraints to make errors tend to zero.

3. CONTROLLER DESIGN BASED ON RHOMBIC INPUT CONSTRAINTS

To design the controller, we need to use the following two lemmas:

Lemma 1 *[7].* f:$[0,\infty) \to R$ is first-order continuous differentiable and $\lim_{t\to\infty} f(t)$ is a finite value, if $\dot{f}(t), t\in[0,\infty)$ is uniformly continuous, then $\lim_{t\to\infty}\dot{f}(t) = 0$.

Lemma 2 *[7].* There is a scalar function $\rho(x)$, $x \in [0, \infty]$, which satisfies the following properties:

(1) $\rho(x)$ is a continuous and non-decreasing function;

(2) $\rho(0) = 0$, and $0 < \rho(x) \le 1$ for $x \in [0, \infty]$;

(3) $\lim_{x\to 0^+} \rho'(x) = \rho_0$, which ρ_0 is a positive constant.

Define $\psi(x)$ as

$$\psi(x) = \begin{cases} \rho(x)\,|\,x & x\in(0,\infty) \\ \rho_0 & x = 0 \end{cases} \tag{10}$$

Then, for $\forall\sigma \in (0, \infty)$, there always exist α and β, such that $\alpha < \psi(x) \le \beta$ holds for $x \in [0, \sigma]$, where both α and β are positive constants.

$\rho(x) = \tanh(x)$ is a function that satisfies the above conditions.

In this paper, we refer to the controller designed in Blažič [8] as follows:

$$\begin{aligned} v &= v_r\cos\theta_e + k_x x_e \\ \omega &= \omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e} + k_\theta\theta_e \end{aligned} \tag{11}$$

where k_x, k_y and k_θ are positive constants. If the errors are too large, then v and ω are more likely to break through the range of the rhombic input area through analysis formula (11). In this way, the control commands cannot be executed well.

Lemma 3 *[7].* For controller (11), if following conditions are met:

(1) $\underline{k}_x \le k_x \le \bar{k}_x, \underline{k}_y \le k_y \le \bar{k}_y, \underline{k}_\theta \le k_\theta \le \bar{k}_\theta$

(2) k_y is differentiable and $\dot{k}_y \ge 0$.

where $\underline{k}_x, \overline{k}_x, \underline{k}_y, \overline{k}_y, \underline{k}_\theta, \overline{k}_\theta$ are positive constant values. Then, trajectory tracking errors of DWMRs will converge to zero, that is x_e, y_e, θ_e will converge to zero.

To use the vector method to design controller (11), we first need to define the controller v and ω as a vector

$$\overrightarrow{OD} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

then by defining other vectors as:

$$\overrightarrow{OA} = \begin{bmatrix} v_r \cos\theta_e \\ \omega_r \end{bmatrix}, \overrightarrow{AB} = \begin{bmatrix} 0 \\ k_y v_r y_e \dfrac{\sin\theta_e}{\theta_e} \end{bmatrix}$$
$$\overrightarrow{BC} = \begin{bmatrix} k_x x_e \\ 0 \end{bmatrix}, \quad \overrightarrow{CD} = \begin{bmatrix} 0 \\ k_\theta \theta_e \end{bmatrix} \tag{12}$$

the controller can be represented by a combination of several vectors:

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} \tag{13}$$

It is necessary to design each vector in turn, so that controller can finally meet the rhombic input constraints.

$$\left|\frac{v_r \cos\theta_e}{m}\right| + \left|\frac{\omega_r l}{m}\right| \le \left|\frac{v_r}{m}\right| + \left|\frac{\omega_r l}{m}\right| \le 1 - \frac{l\varepsilon}{m} \tag{14}$$

From formula (14) we know that $\overrightarrow{OA}$ satisfies the rhombic input constraints, without loss of generality, we represent $\overrightarrow{OA}$ as shown in Figure 3, and because the length of $\overrightarrow{AB}$ is proportional to k_y, we can definitely find a k_y to make $\overrightarrow{AB}$ within the rhombic input constraints. Similarly, we can also find suitable k_x and k_θ so $\overrightarrow{BC}$ and $\overrightarrow{CD}$ can meet the rhombic constraints respectively. Obviously, the controller $\overrightarrow{OD}$ will definitely meet the rhombic input constraints.

Since the requirement for k_y is $\dot{k}_y \ge 0$, we intuitively thought of designing if from Lyapunov function $V(t)$.

$$V(t) = \frac{1}{2}\left(x_e^2 + y_e^2 + \frac{\theta_e^2}{k_y}\right) \tag{15}$$

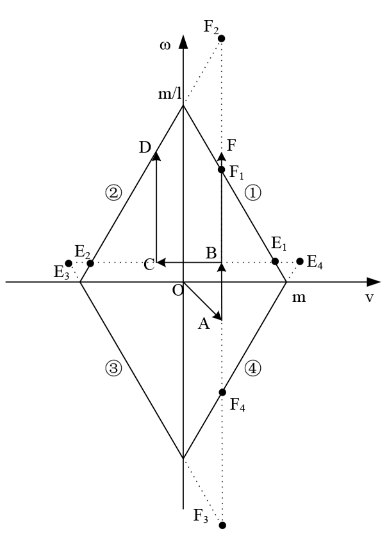

Figure 3 | Vector method design controller.

Let k_y be

$$k_y = \frac{\lambda\varepsilon}{m\sqrt{2V(t) + \mu^2}}$$
$$k_y = \frac{-m\theta_e^2 + \sqrt{m^2\theta_e^4 + 4\lambda^2\varepsilon^2(x_e^2 + y_e^2 + \mu^2)}}{2m(x_e^2 + y_e^2 + \mu^2)} \tag{16}$$

where λ and μ are constants, $0 < \lambda < 1$ and $\mu > 0$.

According to Equations (15) and (16), we can get

$$k_y = \frac{-m\theta_e^2 + \sqrt{m^2\theta_e^4 + 4\lambda^2\varepsilon^2(x_e^2 + y_e^2 + \mu^2)}}{2m(x_e^2 + y_e^2 + \mu^2)} > 0 \tag{17}$$

$$\dot{k}_y = \frac{2k_x k_y^2 x_e^2 + 2k_\theta k_y \theta_e^2}{2k_y(x_e^2 + y_e^2 + \mu^2) + \theta_e^2} \tag{18}$$

If $k_x > 0$ and $k_\theta > 0$, then according to formula (17) and (18), $\dot{k}_y < 0$ can be derived, and further from formula (15) we can get

$$\underline{k}_y = \frac{\lambda\varepsilon}{m\sqrt{2V(0) + \mu^2}} \le k_y \le \frac{\lambda\varepsilon}{m\mu} = \overline{k}_y \tag{19}$$

In this way, the vector $\overrightarrow{OB}$ can be expressed as:

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \left(v_r \cos\theta_e, \omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)^T \tag{20}$$

We can verify that $\overrightarrow{OB}$ satisfies the rhombic input constraints through formulas (14)–(16). Because of k_x, $k_\theta > 0$, so the directions of the vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$ are determined by the signs of x_e and θ_e. We want to occupy the entire area as much as possible under the premise that the controller meets the rhombic input constraints. First, we need to determine the triangle area ΔBEF where the points of C and D are located, as shown in Figure 3, when $x_e < 0$ and $\theta_e > 0$, we take the constraint segment ② to determine the reference triangle ΔBE_2F_2, similarly, when $x_e > 0$ and $\theta_e > 0$, we take the constraint segment ① to determine the reference triangle ΔBE_1F_1, when $x_e < 0$ and $\theta_e < 0$, we get the reference triangle ΔBE_3F_3, and when $x_e < 0$ and $\theta_e < 0$, we get the reference triangle ΔBE_4F_4. Through the formulas of the four constraint lines and the coordinates of point B, we can easily obtain the coordinates of point E as:

$$E: \left(\operatorname{sgn}(x_e)\left(m - \operatorname{sgn}(\theta_e)\left(\omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)l\right),\right.$$
$$\left. w_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right) \tag{21}$$

Similarly, we can get the coordinates of F as:

$$F: \left(v_r \cos\theta_e, \frac{\operatorname{sgn}(\theta_e)(m - \operatorname{sgn}(x_e)v_r \cos\theta_e)}{l}\right) \tag{22}$$

where $\operatorname{sgn}(\cdot)$; is sign function

$$\operatorname{sgn}(x)\begin{cases} |x|/x & x \ne 0 \\ 0 & x = 0 \end{cases} \tag{23}$$

Further we can get the expressions of $\overrightarrow{BE}$ and $\overrightarrow{BF}$ as

$$\begin{aligned}\overrightarrow{BE} &= \overrightarrow{OE} - \overrightarrow{OB} \\ &= \left(\operatorname{sgn}(x_e)(m - \operatorname{sgn}(\theta_e)\left(\omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)l\right) \\ &\quad -v_r \cos\theta_e, 0)^T\end{aligned} \tag{24}$$

$$\begin{aligned}\overrightarrow{BF} &= \overrightarrow{OF} - \overrightarrow{OB} \\ &= \left(0, \frac{\operatorname{sgn}(\theta_e)(m - \operatorname{sgn}(x_e)v_r \cos\theta_e)}{l} - \omega_r - k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)^T\end{aligned} \tag{25}$$

To design k_x and k_θ, let

$$\begin{aligned}\overrightarrow{BC} &= \frac{\rho(|x_e|)}{2}\overrightarrow{BE} \\ \overrightarrow{CD} &= \frac{\rho(|\theta_e|)}{2}\overrightarrow{BF}\end{aligned} \tag{26}$$

Then, we get from (12), (24), (26), that

$$\begin{aligned}k_x &= \frac{\psi(|x_e|)}{2}\left(m - \operatorname{sgn}(\theta_e)\left(\omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)l - \operatorname{sgn}(x_e)v_r\cos\theta_e\right) \\ k_\theta &= \frac{\psi(|\theta_e|)}{2}\left(\frac{m - \operatorname{sgn}(x_e)v_r\cos\theta_e}{l} - \operatorname{sgn}(\theta_e)\left(\omega_r + k_y v_r y_e \frac{\sin\theta_e}{\theta_e}\right)\right)\end{aligned} \tag{27}$$

By formulas (15), (16), (20) and *Lemma 2* we can easily get

$$\begin{aligned}\underline{k}_x &\le \frac{\alpha(1-\lambda)\varepsilon l}{2} \le k_x \le \beta m \le \overline{k}_x \\ \underline{k}_\theta &\le \frac{\alpha(1-\lambda)\varepsilon}{2} \le k_\theta \le \frac{\beta m}{2} \le \overline{k}_\theta\end{aligned} \tag{28}$$

At this point, the k_x, k_y and k_θ meet the two conditions in *Lemma 3*, so the system error will converge to zero. And because of our vector method design the parameters ensure that the control variables v and ω meet the rhombic input constraints too.

4. SIMULATION RESULTS

In this section, we verify the performance of the controller through simulation and compare it with the controller in Chen et al. [7]. Before the simulation starts, some parameters are set as follows:

The maximum velocity of the drive wheels is set to $m = 0.4$ m/s, the wheel spacing is set to $l = 0.16$ m, for setting some parameters of the controller, we choose $\rho(x) = \tanh(x)$, $\varepsilon = 0.1$, $\lambda = 0.99$, $\mu = 0.01$.

Figure 4 shows a robot gradually tracks on the reference trajectory, where the blue line represents the expected trajectory, and the red line represents the actual trajectory. Figure 5 shows the tracking errors x_e, y_e, θ_e are each gradually converge to zero, also we can guarantee the control variables v and ω satisfy the rhombic input constraints through Figure 6, and sometimes v can basically reach the bounds of rhombic input constraints. Figure 7a is the tracking errors diagram under the controller in Chen et al. [7], Figure 7b is

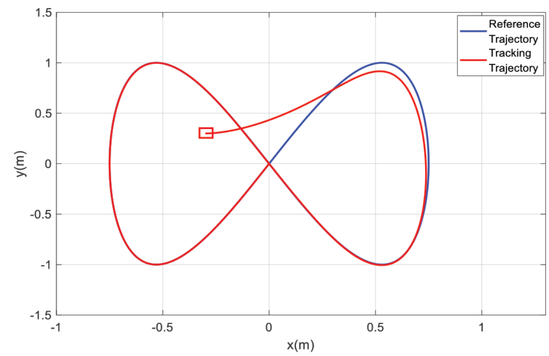

Figure 4 | Tracking reference trajectory.

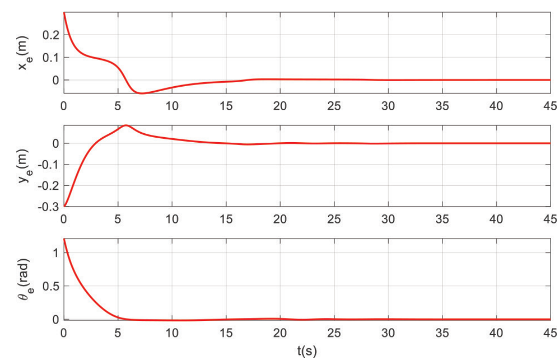

Figure 5 | Tracking errors.

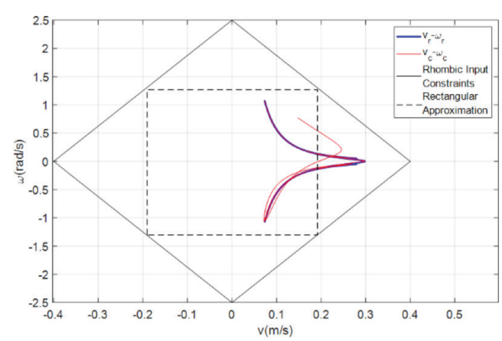

Figure 6 | Input and constraints.

a

b

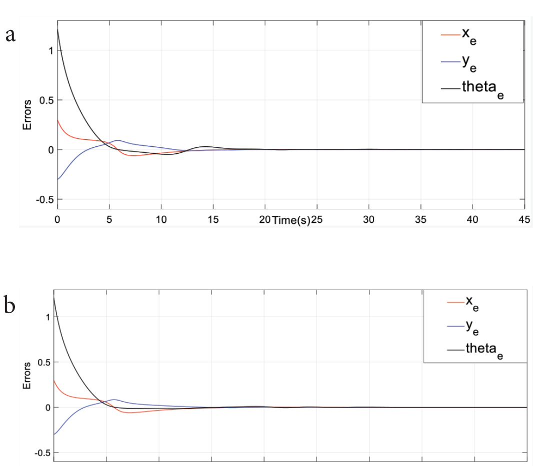

Figure 7 | Controller errors comparison.

the tracking errors diagram under our controller, it can be seen that our controller can make the errors converge faster, and the oscillation is smaller.

5. CONCLUSION

The tracking control problem of DWMRs with rhombic input constraints is solved in this paper. Compared with existing methods, we have improved the design of controller parameters and achieved better performance. Also our method can better exert the robots' mobility and makes the tracking errors converge faster. The controller simultaneously solves the tracking problem and stability problem, its effectiveness can be confirmed by simulation results. Future work will focus on the controller design with uncertainty based on a more complex application environment.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

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