

Research Article**Efficiency Problem of Spherical Robot in Transfer Kinetic Energy**Kenji Kimura¹, Kazuo Ishii²¹Department of Control Engineering, Matsue College of Technology, 14-4, Nishi-ikumacho, Matsue, Shimane, 690-8518, Japan²Graduate School of Life Science and engineering, Kyusyu Institute of Technology, 2-4 Hibikino, Wakamatsu-ku, Kitakyushu 808-0196, Fukuoka, Japan**ARTICLE INFO****Article History**

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ABSTRACT

Previous spherical mobile robots were driven by two rollers with a fixed rotational axis, which restricts the angular velocity vector of the sphere to two dimensions. Three-dimensional freedom is expected to improve the rotational diversity of the sphere. This study proposes a spherical mobile robot with a variable roller-rotational axis that allows the movement of three degrees of freedom. Furthermore, the kinetic energy of transporting the sphere by the rollers is minimized by an optimization procedure and it is shown that the kinetic energy of sphere transport is efficient.

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[\(http://creativecommons.org/licenses/by-nc/4.0/\)](http://creativecommons.org/licenses/by-nc/4.0/).**1. Introduction**

Many types of robots, such as omnidirectional mobile robots and sphere-transported robots, are based on spherical motions. Therefore, various roller arrangements have been proposed for mobile robot applications.

Table 1 shows the usage statuses of single spherical robots operated by different mechanisms, and the dimensions of the existence spaces of their angular velocity vectors. **Figure 1** shows the numbers of rollers N_w arranged per wheel in each mechanism, and their contact types.

In the ACROBAT-S [1] mechanism with $N_w = 2$, the caster of each sphere is driven by two roller drives (see **Figure 1** (a)). The wheel chair mechanism [2] has three spheres (**Figure 1** (b)). The rollers are arranged on the equator, generate an angular velocity vector on the horizontal plane, and can move in all directions. The angular velocity vector of the sphere has two-dimensional freedom. This situation is theoretically

considered in [3]. In the ball dribbling mechanism [4], the rollers are arranged in the upper hemisphere, where they hold the balls by friction (See **Figure 1** (c)). The two-dimensionality of the sphere's angular velocity vector in these mechanisms was affirmed in kinematic studies of a sphere with slipping [5]. In addition, the kinematics of the reverse motion was verified in a demonstration experiment [6].

Among the three-roller cases ($N_w = 3$), OWMPs [7] deployed in highway maintenance move the spheres within three constrained rollers (See **Figure 1** (d)). The rollers are arranged on the equator parallel to the horizontal plane, and the sphere can be rotated in all directions by generating an angular velocity vector on the plane (note that the existence space of the angular velocity vector is two-dimensional).

A ball-balanced robot [8] has three unconstrained rollers (omni-rollers) arranged in a regular triangular configuration (See **Figure 1** (e)). CPU-Ball Bot [9] has four unconstrained rollers fixed in a square configuration (See **Figure 1** (f)).

In the OWMP, A ball-balanced and CPU-Ball Bot mechanisms [7]-[9], the wheel stands are spherical. In particular, A ball-balanced [8] and CPU-Ball Bot [9] mechanisms can rotate the sphere in three-dimensions using the passive rotation characteristics of the non-rotating direction of the omni-roller.

Such three-dimensional freedom of the angular velocity vector of a sphere will improve the motion diversity of the sphere.

However, unconstrained rollers can slip, degrading the accuracy of the control. Therefore, the use of constraining rollers is desired.

As shown in Figure 1 (c), this problem can be solved by freeing the rotation axis of the constraining roller, enabling three-dimensional rotation of the sphere.

Table 1 Existence space of angular velocity vector for sphere mobile robot

| Mechanism | statuses | Rotational Dimension |
|------------------------------|-------------------|----------------------|
| ACROBAT-S[1] | Caster | 2 |
| Wheel chair [2] | Wheel | 2 |
| Ball dribbling mechanism [4] | Sphere conveyance | 2 |
| OWMPs [7] | Wheel | 2 |
| A ball-balanced robot [8] | Wheel | 3 |
| CPU-Ball Bot [9] | Wheel | 3 |

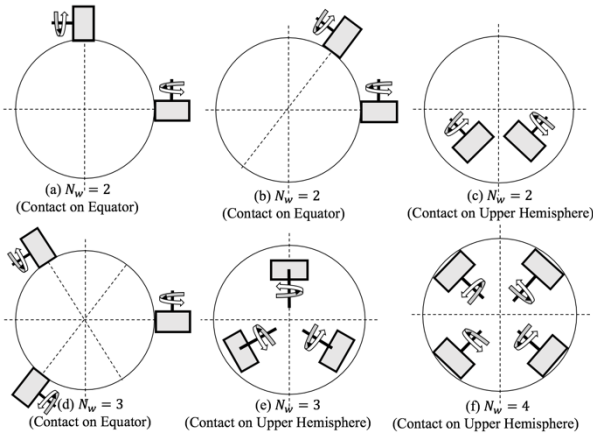


Figure 1 Type of roller arrangement for sphere mobile robot

the angular velocity vector of a sphere will improve the motion diversity of the sphere. However, a mechanism adapting two constrained rollers is suitable in a spherical

object conveyance. It is desired to transport with high kinetic energy efficiency.

In this study, we optimize the total kinetic energy of two rollers. Section 2 calculates the angle of the rotational axis of the sphere, and theoretical formula of minimizes the sum of the kinetic energies of the two rollers. Section 3 presents a simulation of total kinetic energy, and Section 4 summarizes the results and suggests ideas for future work.

2. Total kinetic energy

2.1 Angular velocity vector of the sphere

The center \mathbf{O} of a sphere with radius r is fixed as the origin of the coordinate system $\Sigma - xyz$. The i^{th} constraint roller ($i = 1$ or 2) is in point contact with the sphere at a position vector \mathbf{P}_i and is arranged such that the center of mass of the roller \mathbf{P}_i and \mathbf{O} are on the same line. $\boldsymbol{\omega}$ denotes the angular velocity vector of the sphere. $\boldsymbol{\eta}_i$ denotes the unit vector along the rotational axis of constraint roller. sphere direction φ ($0^\circ \leq \varphi < 360^\circ$) is the angle from x -axis.

Now, given the sphere mobile velocity \mathbf{V} (the center velocity of sphere)

$$\mathbf{V} = \|\mathbf{V}\| [\cos \varphi \quad \sin \varphi \quad 0]^T \quad (1)$$

$\hat{\boldsymbol{\omega}}$ denote orthogonal projection of $\boldsymbol{\omega}$ with respect to xy -plane. $\hat{\boldsymbol{\omega}}$ is perpendicular to \mathbf{V} and represented as Equation (2) (\mathbf{V} is depend on $\hat{\boldsymbol{\omega}}$).

$$\hat{\boldsymbol{\omega}} = \frac{\|\mathbf{V}\|}{r} [-\sin \varphi \quad \cos \varphi \quad 0]^T \quad (2)$$

And, $\hat{\boldsymbol{\omega}}$ is orthogonal projection of $\boldsymbol{\omega}$ with respect to xy -plane. The angle of sphere rotational axis ρ ($-90^\circ \leq \rho \leq 90^\circ$) is the angle between $\boldsymbol{\omega}$ and $\hat{\boldsymbol{\omega}}$. Therefore, $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ has one-dimensional of freedom with parameter ρ ($\omega_z = \|\mathbf{V}\| \tan \rho / r$).

$$\boldsymbol{\omega} = \frac{\|\mathbf{V}\|}{r} [-\sin \varphi \quad \cos \varphi \quad \tan \rho]^T \quad (3)$$

Generally, these depend on the orthogonal projection vector of \mathbf{V} , it has one-dimensional of freedom with respect to $\omega_z = \|\mathbf{V}\| \tan \rho / r$.

2.2 Kinetic energy of the two rollers

Consider two rollers with radius R , moment I , and roller's angular velocity ω_i at \mathbf{P}_i . The summed kinetic energy of the rollers is given by Eqs.(4).

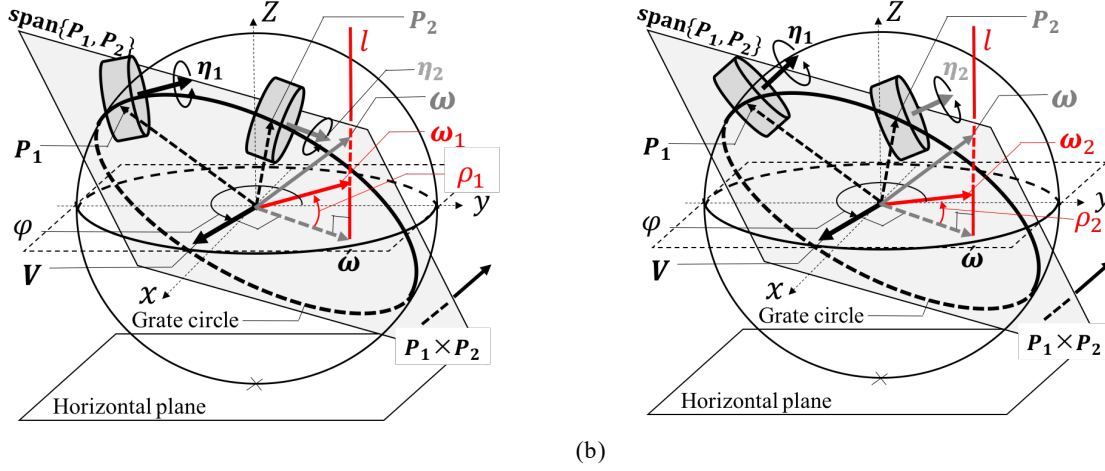


Figure. 2 Existence of the sphere angular velocity vector with respect to sphere mobile velocity.(a) ω_1 is determined (ω_1 is lay on intersection $\text{span}\{P_1, P_2\}$ and line l . (b) ω_2 is determined as minimizes the summed kinetic energies of the two rollers.

$$E = I(\omega_1^2 + \omega_2^2) = \frac{I}{R^2} (\|\omega \times P_1\|^2 + \|\omega \times P_2\|^2) \quad (4)$$

Using the angular velocity vector of a sphere, we now derive the kinetic energy of **Type-I** (with both rotational axes fixed on the same plane) and **Type-II** (with variable rotational axes) configurations(indicated in **Figure 2** (a)(b)).

(A) Case of Type- I

As shown in **Figure 2** (a), if both rotational axes lie on the same plane, omnidirectional locomotion is possible (see [4]). Therefore, the end point of ω_1 can be determined as the intersection of l (the line perpendicular to the horizontal plane passing through end point $\dot{\omega}$) and $\text{span}\{P_1, P_2\}$. The angle of the sphere's rotational axis ρ_1 is then obtained as follows:

$$\rho_1 = \tan^{-1} \left[\frac{(P_1 \times P_2)_x \sin \varphi - (P_1 \times P_2)_y \cos \varphi}{(P_1 \times P_2)_z} \right] \quad (5)$$

By substituting Eqs. (5) into Eqs. (3), we obtain ω_1 as Eqs. (6).

$$\omega_1 = \quad (6)$$

$$\frac{\|V\|}{r} \left[-\sin \varphi, \cos \varphi, \frac{(P_1 \times P_2)_x \sin \varphi - (P_1 \times P_2)_y \cos \varphi}{(P_1 \times P_2)_z} \right]^T$$

Substituting Eqs.(6) into Eqs.(4), we obtain $E = E_1$ as Eqs.(7).

$$E_1 = \frac{I}{R^2} (\|\omega_1 \times P_1\|^2 + \|\omega_1 \times P_2\|^2) \quad (7)$$

(B) Case of Type-II

As shown in **Figure 2** (b), we determine $\rho = \rho_1$, $\omega = \omega_2$ that minimizes the summed kinetic energies of the two rollers, and calculate the minimum energy ($E = E_2$).

To this end, we first express ω as the sum of $\dot{\omega}$ and $\omega_z e_3$.

$$\omega = \dot{\omega} + \omega_z e_3 \quad (8)$$

where

$$e_3 = [0, 0, 1]^T, \quad \dot{\omega} = [\omega_x, \omega_y, 0]^T \quad (9)$$

And,

$$\omega \times P_i = (\dot{\omega} + \omega_z e_3) \times P_i \quad (10)$$

$$= \dot{\omega} \times P_i + \omega_z (e_3 \times P_i)$$

$$\|\omega \times P_i\|^2 \quad (11)$$

$$= \langle \dot{\omega} \times P_i + \omega_z (e_3 \times P_i), \dot{\omega} \times P_i + \omega_z (e_3 \times P_i) \rangle$$

$$= \omega_z^2 \|e_3 \times P_i\|^2 + 2\omega_z \langle \dot{\omega} \times P_i, e_3 \times P_i \rangle + \|\dot{\omega} \times P_i\|^2$$

Using, Eqs.(11), it is represented as quadratic function with respect to ω_z as Eqs.(12)

$$\|\omega \times P_i\|^2 + \|\omega \times P_i\|^2 = \quad (12)$$

$$\begin{aligned}
& (\|e_3 \times P_1\|^2 + \|e_3 \times P_2\|^2)\omega_z^2 \\
& + 2(\langle \dot{\omega} \times P_1, e_3 \times P_1 \rangle + \langle \dot{\omega} \times P_2, e_3 \times P_2 \rangle) \omega_z \\
& + \|\dot{\omega} \times P_1\|^2 + \|\dot{\omega} \times P_2\|^2
\end{aligned}$$

Hence, focusing the coefficients of Eqs.(12), E takes minimal value E_2 as Eqs.(13).

$$\begin{aligned}
E_2 &= (\|\dot{\omega} \times P_1\|^2 + \|\dot{\omega} \times P_2\|^2) \frac{I}{R^2} \\
& - \frac{(\langle \dot{\omega} \times P_1, e_3 \times P_1 \rangle + \langle \dot{\omega} \times P_2, e_3 \times P_2 \rangle)}{\|e_3 \times P_1\|^2 + \|e_3 \times P_2\|^2}
\end{aligned} \quad (13)$$

where

$$\omega_z = - \frac{\langle \dot{\omega} \times P_1, e_3 \times P_1 \rangle + \langle \dot{\omega} \times P_2, e_3 \times P_2 \rangle}{\|e_3 \times P_1\|^2 + \|e_3 \times P_2\|^2} \quad (14)$$

$$\rho_2 = \tan^{-1} \left[- \frac{\langle \dot{\omega} \times P_1, e_3 \times P_1 \rangle + \langle \dot{\omega} \times P_2, e_3 \times P_2 \rangle}{\|e_3 \times P_1\|^2 + \|e_3 \times P_2\|^2} \right] \quad (15)$$

Substituting Eqs.(15) into Eqs.(3), ω_2 as Eqs.(16).

$$\omega_2 = \quad (16)$$

$$\frac{\|V\|}{r} \left[-\sin \varphi, \cos \varphi, - \frac{\langle \dot{\omega} \times P_1, e_3 \times P_1 \rangle + \langle \dot{\omega} \times P_2, e_3 \times P_2 \rangle}{\|e_3 \times P_1\|^2 + \|e_3 \times P_2\|^2} \right]^T$$

3. Simulation

This section presents the simulation results ω_1 (See Eqs.(6)), ω_2 (See Eqs.(16)), E_1 (See Eqs.(7)) and E_2 (See Eqs.(13)) with parameter φ ($0^\circ \leq \varphi < 360^\circ$) in the given sphere mobile speed: $\|V\| = 1$ [m/s]. The conditions are as follows: $I = 1$, $R = 0.1$ [m], $\|V\| = 1$ [m/s], $r = 0.1$ [m], $\theta_{1,1} = 215^\circ$, $\theta_{1,2} = 60^\circ$, $\theta_{2,1} = 325^\circ$, $\theta_{2,2} = 60^\circ$.

3.1 Trajectory of the end point of the angular velocity vector and Totally kinetic energy

This section presents the simulated trajectory of the end points of the angular velocity vectors ω_1 and ω_2 , and the kinetic energies E_1 and E_2 .

As shown in **Figure 3** (a), the ellipsoid trajectory of ω_2 lies nearer to the xy -plane than that of ω_1 , and both trajectories cross a common line parallel to the y -axis.

As shown in **Figure 3** (b), E_1 is minimized at sphere direction angles $\varphi = 90^\circ$ and 270° , and maximized at $\varphi = 0^\circ$ and 180° . Meanwhile, E_2 is minimized at $\varphi = 0^\circ$ and 180° , and maximized at $\varphi = 90^\circ$ and 270° . And, E_1 and E_2 are same value at $\varphi = 90^\circ$ and 270° .

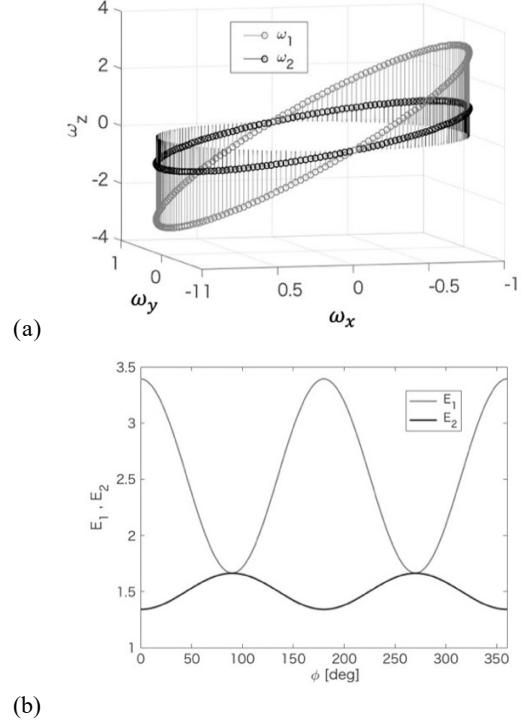


Figure 3 Simulation result. (a) Trajectory of angular velocity vector of sphere. (b) Totally kinetic energy

3.2 Example of sphere transfer

This subsection simulates the sphere transportation process. As shown in **Figure 4**, the robot's transfer route starts at O and reaches P and Q . Each path OP ($\varphi = 90^\circ$), PQ ($\varphi = 45^\circ$) and QR ($\varphi = 0^\circ$) is completed after some time t ($0 \leq t \leq 5$).

The direction angle and mobile speed of the spherical wheel are defined as functions of t (see **Figure 5**).

$$\varphi(t) = \begin{cases} 90^\circ & 0 \leq t \leq 5 \\ 45^\circ & 5 \leq t \leq 10 \\ 0^\circ & 10 \leq t \leq 15 \end{cases} \quad (17)$$

$$\|V\| = \begin{cases} t & (0 \leq t \leq 1, 5 \leq t \leq 6, 10 \leq t \leq 11) \\ 1 & (1 \leq t \leq 4, 6 \leq t \leq 9, 11 \leq t \leq 14) \\ 5 - t & (4 \leq t \leq 5, 9 \leq t \leq 10, 14 \leq t \leq 15) \end{cases} \quad (18)$$

Substituting Eqs. (17) and (18) into Eqs. (6) and (16) respectively, we obtain $\omega_1(t)$ and $\omega_2(t)$, respectively. Moreover, substituting $\omega_1(t)$ and $\omega_2(t)$ into Eqs. (7) and (13) respectively, we obtain $E_1(t)$ and $E_2(t)$, respectively.

The simulation result is shown in Figure 6. In the interval $[0, 5]$, the robot is traveling straight, so we have $E_1(t) = E_2(t)$. In the interval $[5, 15]$, $E_2(t)$ is smaller than $E_1(t)$.

Table 2 The totally kinetic energy efficiency in sphere transport

| Migration pathway | <i>OP</i> | <i>PQ</i> | <i>QR</i> | |
|-------------------|-------------------------------------|------------|------------|------|
| $[a, b]$ | $[0, 5]$ | $[5, 10]$ | $[10, 15]$ | |
| φ | 90° | 45° | 0° | |
| $E_i^{[a,b]}$ | Type-I | 1.66 | 2.55 | 3.39 |
| | Type-II | 1.66 | 1.49 | 1.34 |
| $\gamma^{[a,b]}$ | 0 % | 41 % | 61 % | |
| $\gamma^{[0,15]}$ | 34 % (Total energy reduction ratio) | | | |

And, we define integration of $E_1(t), E_2(t)$ with respect to time interval $[a, b]$.

$$E_i^{[a,b]} = \int_a^b E_i(t) dx \quad (19)$$

Energy-reduction ratio in $[a, b]$ is defined as follow.

$$\gamma^{[a,b]} = 100(1 - E_2^{[a,b]} / E_1^{[a,b]}) [\%] \quad (20)$$

As shown in Table 2, it is represented by $\gamma^{[0,5]} = 0 [\%], \gamma^{[5,10]} = 41 [\%], \gamma^{[10,15]} = 61 [\%]$ and is $\gamma^{[0,15]} = 34 [\%]$. Therefore, the kinetic energy of sphere transport was efficient.

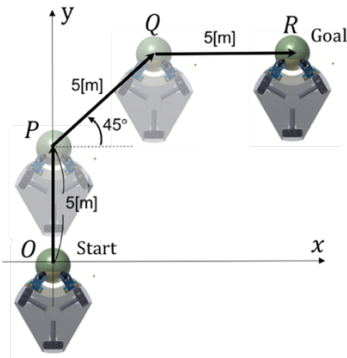
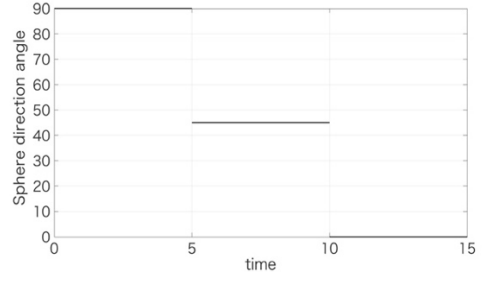
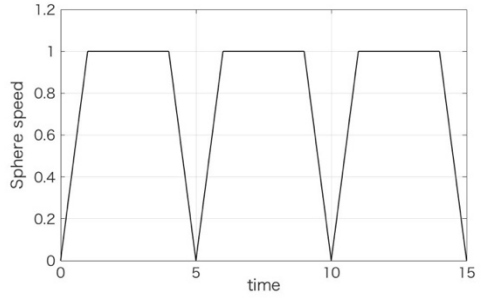


Figure 4 The robot's transfer route



(a)



(b)

Figure 5 The definition (a). Sphere direction (b). Sphere mobile speed

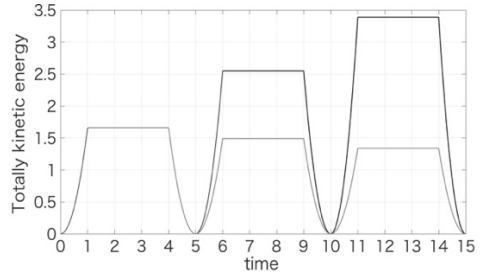


Figure 6 Simulation result of totally kinetic energy

4. Conclusion

We proposed a non-offset mechanism with three-dimensional freedom under the assumption of variable rotational axes of the rollers. The kinetic energy consumption was compared with that of the previous fixed type in a simulation of sphere transportation, which confirmed a large energy reduction in the proposed mechanism.

In future work, we plan to evaluate the kinetic energy of the roller arrangement at an arbitrary contact point on the upper hemisphere as an evaluation function.

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Authors Introduction

Dr. Kenji Kimura



He is a Lecturer in National Institute of Technology, Matsue College, where he has been since 2022. He received the ME (mathematics) from Kyusyu University in 2002 and received his Ph.D. degree in engineering from Kyushu Institute of in 2020.

Then, He was mathematical teacher of International Baccalaureate Diploma Program (Mathematics) and engineering course chief in Fukuoka Daiichi High School, His research interests spherical mobile robot kinematics, control for object manipulation.

Dr. Kazuo Ishii



He is a Professor in the Kyushu Institute of Technology, where he has been since 1996. He received his Ph.D. degree in engineering from University of Tokyo, Japan, in 1996. His research interests span both ship marine engineering and Intelligent Mechanics. He holds five patents derived from his research. His lab got "Robo Cup 2011 Middle Size League Technical Challenge 1st Place" in 2011. He is a member of the Institute of Electrical and Electronics Engineers, the Japan Society of Mechanical Engineers, Robotics Society of Japan, the Society of Instrument and Control Engineers and so on.