SUGISAKA MASANORI

Journal of Robotics, Networking and Artificial Life
Vol. 9(1); June (2022), pp. 72–76
ISSN (Online): 2352-6386; ISSN (Print): 2405-9021
https://alife-robotics.org/jrnal.html

Research Article

Bipartite Consensus of Linear Discrete-time Multi-agent Systems with Encoding-Decoding

Jintao Hu, Yingmin Jia, Yaxin Li
The Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University (BUAA), 37 Xueyuan Road, Haidian District, Beijing, 100191, China

ARTICLE INFO

Article History

Received 20 November 2020
Accepted 10 August 2021

Keywords

Multi-agent systems
signed undirected graph
bipartite consensus
structural balance/unbalance
encoding and decoding

ABSTRACT

In this paper, the consensus problem of general linear discrete-time multi-agent systems with the cooperative-antagonistic interactions and limited communication bandwidth is investigated, and a bipartite consensus con-trol protocol is proposed for each agent, which is given in terms of the state of its dynamic encoder and decod-ers with uniform quantizer. It's demonstrated that the bipartite consensus can be realized, if the signed undi-rected graph is connected and structurally balanced/unbalanced. Furthermore, the explicit form of the conver-gence rate is given. Numerous simulations are presented to illustrate the feasibility of the control scheme used on this system.

1. Introduction

Recently, the consensus problem in the field of multi-agent systems (MASs) has received much attention due to its broad application prospects, including attitude synchronization of spacecraft[1], distributed computation, market and economic theory in the complex environment, self-driven motion of biological particles and wireless multi-sensor network, etc.

In many relevant works, it's assumed that the agents can obtain exact information of the states of the neighbors by local communication. This means that the communication bandwidth is infinite. However, in practice, the communication ability of the network is limited. Thus, the encoding and decoding scheme is introduced to deal with this problem. At each stage, the sender of the agents encodes the quantized state and sends code to neighbors. The receiver of the neighbors gets the code and decodes it to estimate the state of the sender[2-4].

In addition, most of the works in the literature assume that the relationship between agents is cooperative. However, cooperation often coexists with antagonism actually. In this case, the concept of structural balance/unbalance is given. By utilizing the property of structural balance/unbalance of communication network, the bipartite consensus is introduced where all gents attain agreement concerning a value, which is same for all in modulus but not in sign[5-7]. However, there is no research date on the bipartite consensus of MASs with the encoding and decoding mechanism.

Motivated by the above discussion, we consider a consensus control protocol which is designed based on a dynamic encoding-decoding mechanism. The agents can achieve bipartite consensus under the control scheme. Moreover, the clear form of the convergence rate is given.

The following notations will be used in this paper:

Corresponding author's E-mail: hujintao@buaa.edu.cn, ymjia@buaa.edu.cn www.buaa.edu.cn

1_n	n-dimensional column vector whose elements are all 1
$\lVert \cdot \rVert$	Euclidean norm of a vector or matrix
$\lVert \cdot \rVert_\infty$	Infinite norm of a vector or matrix
$\rho(\cdot)$	Spectral radius of a matrix
$\mathrm{sgn}(\cdot)$	Sign function

2. Preliminaries and Problem Formulation

2.1 Signed Graph

Let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be a signed undirected graph, where $\mathcal{V}=\{v_1,v_2,\cdots,v_N\}$ is a set of N agents with i denoting the ith agent, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set of agents, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ with the signed elements is the weighted adjacency matrix of $\mathcal{G}$. The set of agents who can communicate with agent i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}\mid(i,j)\in\mathcal{E}\}$. The Laplacian matrix of $\mathcal{G}$ is represented by $\mathcal{L}=\mathcal{C}-\mathcal{A}$, where $\mathcal{C}=diag(\sum_{j\in\mathcal{N}_1}|a_{1j}|,\cdots,\sum_{j\in\mathcal{N}_N}|a_{Nj}|)$. A signed graph $\mathcal{G}$ is structurally balanced if the nodes in $\mathcal{V}$ are divided into two parts $\{\mathcal{V}_1,\mathcal{V}_2\}$, where $\mathcal{V}_1\bigcup\mathcal{V}_2=\mathcal{V}$ and $\mathcal{V}_1\bigcap\mathcal{V}_2=\varnothing$, bring $a_{ij}\geq 0$ $\forall i,j\in\mathcal{V}_p\left(p\in\{1,2\}\right)$, $a_{ij}\leq 0$ $\forall v_i\in\mathcal{V}_p$, $v_j\in\mathcal{V}_q$, $p\neq q$ $\left(p,q\in\{1,2\}\right)$, otherwise, the signed graph $\mathcal{G}$ is structurally unbalanced.

2.2. Problem Formulation

In this paper, we consider the bipartite consensus control for a multi-agent system with the discrete-time dynamics

$$x_i(k+1)=Ax_i(k)+Bu_i(k) \quad (1)$$

Where $x_i(k)\in\mathbb{R}^n$ and $u_i(k)\in\mathbb{R}$, respectively, represent the state and input of the agent i at time k. $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times 1}$ denote the state and input matrix.

A uniform quantizer is utilized to quantize the state of agent. The quantizer $q(x)$ is defined as follows:

$$q(x)=\begin{cases}0, & -1/2\leq x\leq 1/2\\ b, & (2b-1)/2<x\leq(2b+1)/2,\\ & b=1,\cdots,L-1\\ L, & x>(2L-1)/2\\ -q(-x), & x<-1/2\end{cases} \quad (2)$$

Obviously, the level of the quantizer is $2L+1$. When $|x|\leq L+1/2$, the quantization error of the uniform quantizer satisfies $|x-q(x)|\leq 1/2$. Assume that the information transmitted between agents only can be binary number, but not exact value. Thus, the corresponding encoder and decoder are designed. The encoder of agent i is proposed as follows:

$$\begin{cases}\hat{\phi}_i(k)=l(k-1)Z_i(k)+A\hat{\phi}_i(k-1),\hat{\phi}_i(0)=0\\ Z_i(k)=Q(\dfrac{x_i(k)-A\hat{\phi}_i(k-1)}{l(k-1)})\end{cases} \quad (3)$$

Where $\hat{\phi}_i(k)$ and $Z_i(k)$ are the internal variable and output of the encoder, respectively. Apart from this, $Q(\cdot)=[q(\cdot),\cdots,q(\cdot)]^T$ represents the product quantizer. $l(k)=l_0r^k$ is the scaling function, where $l_0\in\mathbb{R}$ and $r\in(0,1)$, which prevents the quantizer from saturation. The decoder of agent j which receives the information from agent i is designed as follows:

$$\begin{cases}\hat{\varphi}_{ji}(0)=0\\ \hat{\varphi}_{ji}(k)=l(k-1)Z_i(k)+A\hat{\varphi}_{ji}(k-1)\end{cases} \quad (4)$$

Where $\hat{\varphi}_{ji}(k)$ which is connected with agent i is the internal variable of the decoder of agent j.

The objective of this paper is to propose a distributed control scheme based on dynamic encoding and decoding mechanism, such that the multi-agent system realizes bipartite consensus when communication graph is structurally balanced/unbalanced.

For this purpose, we consider the following protocol

$$u_i(k)=K\sum_{j\in\mathcal{N}_i}|a_{ij}|(\hat{\phi}_i(k)-\mathrm{sgn}(a_{ij})\hat{\varphi}_{ij}(k)) \quad (5)$$

Where $K\in\mathbb{R}^{1\times n}$ is the control gain to be designed later.

Definition 1. *The consensus of multi-agent systems can be achieved, if there exists the control law such that*

$$\lim_{k\to\infty}\lVert x_i(k)-\mathrm{sgn}(a_{ij})x_j(k)\rVert=0$$

For any initial state $x_i(0)$, $\forall i\in[1,N]$.

Assumption 1. $\lVert x_i(0)\rVert_\infty\leq C_x,\forall i\in[1,N]$ *and* C_x *can be obtained by all agents.*

Assumption 2. *There exists* $K\in\mathbb{R}^{1\times n}$ *such that the eigenvalues of* $A-\lambda_i(\mathcal{L})BK,i=2,\cdots,N$ *are distributed inside the open unit circle of complex plane.*

3. Main Results

3.1. Structurally balanced graph

To get the main result, we need divide the agents into two groups $d_i\in\{1,-1\}$ $i=1,\cdots,N$, where the agents are in the same group when they are cooperating, i.e., $a_{ij} > 0$.

Lemma 1. *For a connected signed and structurally balanced graph, there always exists a diagonal matrix* $D = diag(d_1, \cdots, d_N)$ *such that all elements of* $D\mathcal{A}D$ *are nonnegative, and* $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \le \cdots \le \lambda_N(\mathcal{L})$.

Lemma 2. *For any* $P \in \mathbb{R}^{n\times n}$ *and* $\varepsilon > 0$, *we can obtain that* $\|P^k\| \le M\eta^k, \forall k \ge 0$, *where* $M = \sqrt{n}(1+2/\varepsilon)^{n-1}$ *and* $\eta = \rho(P) + \varepsilon\|P\|$.

Theorem 1. *Consider the multi-agent system (1) with the structurally balanced and connected signed undirected communication network. Suppose that Assumption 1 and 2 hold. Select appropriate control gain* K *such that* $\rho(J(K)) < 1$, *where* $J(K) = diag(A, A+\lambda_2 BK, \cdots A+\lambda_N BK)$. *For any* $r \in (\rho(J(K)), 1)$ *and* $\sigma \in (0, (r-\rho(J(K))) / \|J(K)\|)$, *let*

$$M(K,r) = \frac{\|A\|_\infty + 2d^* \|BK\|_\infty}{2r} + \frac{n^{3/2}\lambda_N^2 M\sqrt{N}\|BK\|_\infty^2}{r^2(1-\eta)}$$

and for any $L > M(K,r) - 1/2$, *let*

$$l_0 > \max\{\frac{\|A\|_\infty C_x}{L+1/2}, \frac{4r^2 C_x(1-\eta)}{\sqrt{n}\lambda_N \|BK\|_\infty}\}$$

Then under the proposed control scheme (5) given by (2), (3) and (4), the closed-loop system realizes the bipartite average consensus

$$\lim_{k\to\infty} x_i(k) = d_i \sum_{j=1}^{N} d_j x_j(0),\ i = 1, \cdots, N$$

Proof. For the specific proof process, please refer to Theorem 1 in [8].

3.2. Structurally unbalanced graph

In order to deal with the condition of MASs with structurally unbalanced communication graph, we need the following lemma.

Lemma 3. *If a connected signed graph is structurally unbalanced, the corresponding Laplacian matrix* $\mathcal{L}$ *satisfies* $0 < \lambda_1(\mathcal{L}) \le \lambda_2(\mathcal{L}) \le \cdots \le \lambda_N(\mathcal{L})$.

The main result in this subsection is similar to Theorem 1. The key difference is that the signed average state error $\Delta(k)$ is substituted by the state $X(k)$ of MASs, and the conclusion turns into $\lim_{k\to\infty} x_i(k) = \mathbf{0},\ i = 1, \cdots, N$. Besides, because of the structurally unbalanced communication graph, the block diagonal matrix $J(K)$ is changed with $diag(A+\lambda_1 BK, A+\lambda_2 BK, \cdots, A+\lambda_N BK)$. Please refer to theorem 1 for specific proof.

4. Simulation and analysis

Consider a signed connected undirected and structurally balanced communication graph $\mathcal{G}$ with 4 nodes and the adjacency matrix

$$\mathcal{A} = \begin{bmatrix} 0 & 0.2 & -0.2 & -0.2 \\ 0.2 & 0 & -0.1 & 0 \\ -0.2 & -0.1 & 0 & 0.1 \\ -0.2 & 0 & 0.1 & 0 \end{bmatrix}$$

The state and input matrix are assumed to be $A = B = 1$. The initial states are chosen as $x_i(0) = i,\ i = 1, \cdots, 4$ .The control gain K , the initial value l_0 and factor r of scaling function are taken as 2, 4 and 0.995, respectively. Besides, we take $L = 1$. The evolution of the states of these multi-agents is presented in Fig.1. It can be seen that the bipartite average consensus is achieved. Then, we choose $r = 0.99$, and the corresponding result is shown in Fig.2. Combine these two figures, we can find that the smaller the factor r , the faster the state converges.

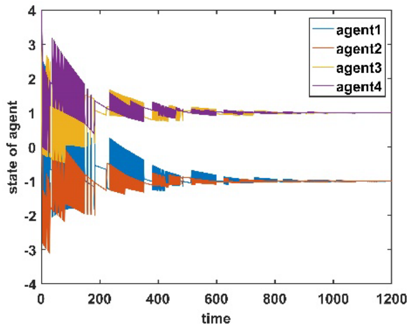

Fig.1. States of the MASs with structurally balanced communication network and $r = 0.995$.

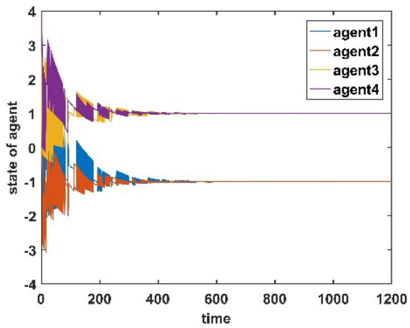

Fig.2. States of the MASs with structurally balanced communication network and $r = 0.99$.

With regard to structurally unbalanced graph $\mathcal{G}$, the adjacency matrix is chosen as

$$\mathcal{A}=\begin{bmatrix} 0 & 0.2 & -0.2 & -0.2 \\ 0.2 & 0 & 0.1 & 0 \\ -0.2 & 0.1 & 0 & 0.1 \\ -0.2 & 0 & 0.1 & 0 \end{bmatrix}$$

The evolution of the states of these multi-agents which eventually converge to zero is shown in Fig.3.

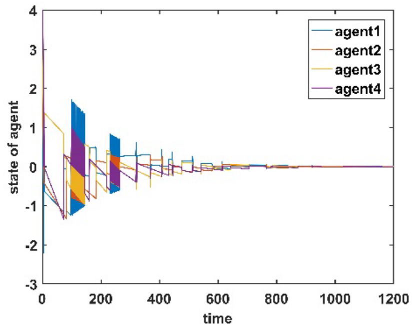

Fig.3. States of the MASs with structurally unbalanced communication network and $r=0.995$.

5. Conclusion

The bipartite consensus control problem of multi-agent systems with limited communication rate based on the signed connected undirected network topology is solved in this paper. For structurally balanced/unbalanced topology, it's demonstrated that the consensus can be realized by employing the proposed control scheme. In addition, the consensus convergence rate is given explicitly. Future research topics about the coding-decoding-based information transmission problem may include these aspects of communication delay, unmeasurable state information, event-triggered sampling and packet loss, etc.

Acknowledgements

This work was supported by the NSFC (62133001, 61520106010) and the National Basic Research Program of China (973 Program: 2012CB821201).

References

1. X. Lu, Y. Jia: Robust Adaptive Attitude Synchronization of Uncertain Rigid Bodies on Special Orthogonal Group with Communication Delays and Gyro Biases , J International Journal of Control Automation and Systems, vol. 17, No.11, pp. 2769-2783, 2019.
2. T. Li, M. Fu, L. Xie, J. Zhang: Distributed Consensus with Limited Communication Data Rate, IEEE Transactions on Automatic Control, vol. 56, No.2, pp.279-292, 2011.
3. K. You, L. Xie: Network Topology and Communication Data Rate for Consensusability of Discrete-Time Multi-agent Systems, IEEE Transactions on Automatic Control, vol. 56, No.10, pp.2262-2275, 2011.
4. Y. Meng, T. Li, J. Zhang: Coordination over Multi-agent Networks with Unmeasurablr States and Finite-Level Quantization, IEEE Transactions on Automatic Control, vol. 62, No.9, pp.4647-4653, 2011.
5. C. Altafini: Consensus Problems on Networks with Antagonistic Interactions, IEEE Transactions on Automatic Control, vol. 58, No.4, pp.935-946, 2013.
6. D. Meng, Y. Jia, J. Du: Finite-Time Consensus for Multiagent Systems with Cooperative and Antagonistic Interactions, IEEE Transactions on Neural Networks and Learning Systems, vol. 27, No.4, pp.762-770, 2016.
7. Y. Zhu, S. Li, J. Ma, Y. Zheng: Bipartite Consensus in Networks of Agents with Antagonistic Interactions and Quantization, IEEE Transactions on Circuits and Systems, vol. 65, No.12, pp.2012-2016, 2018.
8. J. Hu, Y. Jia, Y. Li: Consensus Control of Linear Discrete-time Multi-agent Systems with Limited Communication Data Rate, Proceedings of the 2022 International Conference on Artificial Life and Robotics, pp. 416-419, 2022.

Authors Introduction

Mr. Jintao Hu

He received the B.S. degree in automation from Chongqing University, China, in 2019. He is currently pursuing the Ph.D. degree with the Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University, China. His research interests include consensus control of multi-agent systems and event-triggered control.

Prof. Yingmin Jia

He received the B.S. degree in control theory from Shandong University, China, in 1982, and the M.S. and PhD degrees both in control theory and applications from Beihang University, China, in 1990 and 1993, respectively. Then, he joined the Seventh Research Division at Beihang University where he is currently Professor of automatic control. His research interests include robust control, intelligent control and their applications in robot systems.

Mr. Yaxin Li

He received the B.S. degree in automation from Shandong University, China, in 2018. He is currently pursuing the Ph.D. degree with the Seventh Research Division and the Center for Information and Control, School of Automation Science and Electrical Engineering, Beihang University, China. His research interests include distributed control and formation control of multi-agent systems.