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Research Article

Formation control of rectangular agents using complex Laplacian

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ABSTRACT

A distributed control algorithm based on artificial potential function, coordinate transformation and complex Laplacian for formation control of second-order rectangular agents is presented in this paper. The system can achieve the desired formation, avoid collision and maintain connectivity between agents. The stability analysis of this algorithm is given. Finally, the effectiveness of the algorithm is illustrated by the simulation results..

1. Introduction

Formation control of multi-agent systems has attracted a lot of attention in recent years because of the progress of science and technology, especially the development of artificial intelligence, robots, sensors, and communication technology[1-3].

Researches about formation control mainly focus on formation structure and the stability of the systems[4-6]. Graph theory is usually used to define the positional relationship between agents and the shape of formation. The obstacle avoidance strategy between agents and environmental obstacles is the premise of the stability of multi-agent systems. Artificial potential function is often used to realize collision avoidance and communication maintenance between agents.

The shape of most of the agent models studied are particle or circular. Considering some models in transportation system, rectangular agents are considered in our paper, which can reduce redundancy when we design collision avoidance areas. Formation control and obstacle avoidance of first-order rectangular multi-agent systems have been studied[7]. On this basis, other articles consider the problem of maintaining connectivity[8]. This paper focuses on second-order multi-agent systems and presents the control law to make the systems reach the desired formation, avoid collision and maintain communication.

Notations: *The length of the rectangle agent i is l_i, the width of the rectangle agent i is w_i. L_s denotes the Laplacian matrix of sensing graph. L_c denotes the Laplacian matrix of communication graph.*

2. Preliminaries and Problem Statement

2.1. *Coordinate transformation*

Considering two rectangle agents (agent 1 and agent2) and one obstacle (obs) as shown in Fig. 1, the coordinate of center of rectangle agent 1 is denoted by O_1 (x_1, y_1) and the coordinate of vertex P_{11} is denoted by P_{11} (x_{11}, y_{11}) in the frame OXY, the coordinate of center of rectangle agent 1 is denoted by O^1_1 (x^1_1, y^1_1) and the coordinate of vertex P_{11} is denoted by P^1_{11} (x^1_{11}, y^1_{11}) in the frame $O_1X_1Y_1$. φ_1 is the angle between the X axis and the line between vertex and O_1.

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Denote $P_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, P_{1k} = \begin{bmatrix} x_{1k} \\ y_{1k} \end{bmatrix}$ in the frame OXY, and $P^1{}_{1k} = \begin{bmatrix} x^1_{1k} \\ y^1_{1k} \end{bmatrix}$ in the frame $O_1X_1Y_1$ for k=1,2,3,4,

$$P^1_{1k} = R(\varphi_1)(P_{1k} - P_1) \tag{1}$$

where $R(\varphi_1) = \begin{bmatrix} \cos(\varphi_1) & \sin(\varphi_1) \\ -\sin(\varphi_1) & \cos(\varphi_1) \end{bmatrix}$.

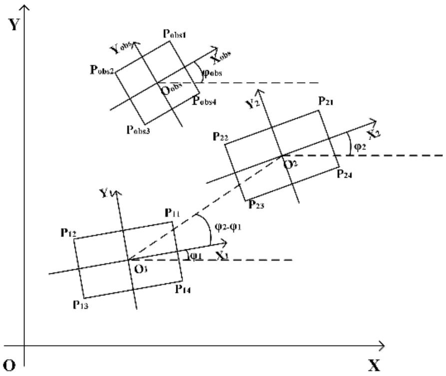

Fig. 1. Coordinate transformation

The distance between two agents is defined as follows.

$$d_{x1}(p_{2k}) = \begin{cases} \left|x^1_{2k}\right| - \dfrac{l_1}{2}, & if \left|x^1_{2k}\right| > \dfrac{l_1}{2} \\ 0, & otherwise \end{cases}$$

$$d_{y1}(p_{2k}) = \begin{cases} \left|y^1_{2k}\right| - \dfrac{w_1}{2}, & if \left|y^1_{2k}\right| > \dfrac{w_1}{2} \\ 0, & otherwise \end{cases}$$

$$d_1(p_{2k}) = \sqrt{d_{x1}(p_{2k})^2 + d_{y1}(p_{2k})^2}$$

Define the distance between agent 1 and agent 2 is $d_{12} = \min\left(\min_k\left(d_1(p_{2k})\right), \min_k\left(d_2(p_{1k})\right)\right)$.

2.2. *Graph theory*

A directed graph $G(v,\varepsilon)$ consists of a node set $v = \{1,2,\cdots n\}$ and a set of edges $\varepsilon \subset v \times v$. If the edge (j,i) is in edge ε, j is the in-neighbor of agent i and i is the out-neighbor of agent j.

For a directed graph, if there exists a path from node q_2 to node q_1, we say that node q_1 is reachable from node q_2. If there exists a node q_i, which is reachable from any other node q_i, we say the directed graph is rooted. Supposed existing a non-singleton subset of nodes Q: $\{Q_1, Q_2, \ldots, Q_n\}$, if there exists a path from nodes $\{Q_1, Q_2, \ldots, Q_n\}$ to q_i after removing any node in Q, we say q_i is 2-reachable from Q. If there exists a subset of two nodes, which is 2-reachable from any other node, we say the directed graph is 2-rooted.

2.3. *Collision and connection region*

Define two areas for obstacle avoidance (Ψ^a_i) and communication maintenance (Ψ^c_i) as shown in Fig. 2. $\Psi^a_i : \left\{ j \in \square : r_a \le \left\| d_{ij} \right\| \le R_a \right\}$ .If $\left\| d_{ij} \right\| < r_a$, there may be collisions between agents. $\Psi^a_i : \left\{ j \in \square : r_m \le \left\| d_{ij} \right\| \le R_m \right\}$. If $\left\| d_{ij} \right\| \ge R_m$, agents will lose contact.

2.4. *Desired formation*

Given a desired relative position formation β : $\{0, \beta_2, \cdots, \beta_n\}, \beta_i \in \square$, if the final position can be expressed as $\lim_{t\to\infty} p(t) = \varpi_1\beta + \varpi_2\mathbf{1_n} + v_n t\mathbf{1_n}$, $\varpi_1, \varpi_2 \in \square$, we say the target formation is formed.

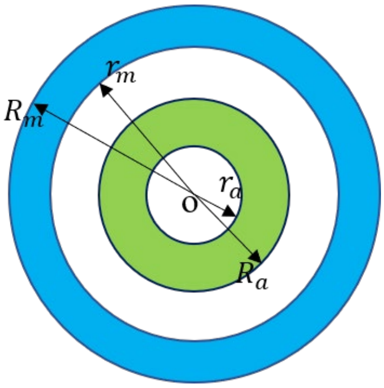

Fig. 2. Communication and collision avoidance area

3. Control Design

Consider a system with n agents, and the dynamic model is as follows:

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i \end{cases} \tag{2}$$

where $p_i \in \square$ donates the position of agent i, $v_i \in \square$ donates the velocity of agent i, $a_i \in \square$ denotes the acceleration of agent i.

Consider the problem: how to design the control protocol to make the system reach the desired formation and avoid collision between agents.

The control protocol is designed as follows:

$$u_i = u_{if} + u_{ic}, i = 1, \dots, n \tag{3}$$

where u_{if} is used for formation maneuvering and u_{ic} is used for collision avoidance.

u_{if} is obtained from the following equation:

$$\begin{cases} \dot{\tilde{v}}_i = \sum_{j \in M_i} a_{ij}\left(\tilde{v}_j - \tilde{v}_i\right) \\ \dot{\tilde{p}}_i = -\sum_{j \in \mathcal{N}_i^+} s_{ij}\left(p_j - p_i\right) - \alpha \tilde{p}_i \\ u_{if} = -\sum_{j \in \mathcal{N}_i^+} s_{ij}^* \tilde{p}_i + \sum_{j \in \mathcal{N}_i^-} s_{ji}^* \tilde{p}_j + \tilde{v}_i - v_i \end{cases} \tag{4}$$

where $\tilde{p}_i$ is an auxiliary variable which is used to form the desired shape, $\tilde{v}_i$ is an auxiliary variable which is used to synchronize the velocity. M_i is the neighbor of agent i in communication graph, N_i^+ is the in-neighbor of agent i in sensing graph, N_i^- is the out-neighbor of agent i in sensing graph. a_{ij} can be any positive real number, $s_{ij} \in$ □ satisfies $\sum_{j \in \mathcal{N}_i^+} s_{ij}(\beta_j - \beta_i) = 0,\ i = 1, \dots, n$ .The variable α satisfies $\alpha > \dfrac{\sqrt{1 + 4\lambda_{\max}\left(L_s^* L_s\right)} - 1}{2}$.

Design a potential function Θ_{ij} :

$$\Theta_{ij} = \begin{cases} \left(\dfrac{\left|p_i - p_j\right|^2}{\sqrt{\tan \mu_1\left(\left|p_i - p_j\right|^2 - r_a\right)}}\right)^{1.4} & ,\text{if } p_j \in \Psi_i^a \\ 0 & ,\text{otherwise} \end{cases}$$

where μ_1 is a constant.

$$\begin{aligned} \vartheta_i &= -\sum_{p_j \in \Psi_i^a} \frac{\partial \Theta_{ij}}{\partial p_i},\ i = 1, \dots, n \\ u_{ic} &= \left(\left|u_{if}\right| + \mu_3\right) \operatorname{sgn}\left(\vartheta_i\right) \end{aligned} \tag{5}$$

where μ_3 is a constant.

Theorem 1. *Using control protocols in Equations (3), (4) and (5), the multi-agent systems can achieve the desired formation and avoid collision.*

Proof. The proof is similar to Theorem 3.1 in [9]. □

Consider that how to design the control law to make the system reach the desired formation, avoid collision and maintain communication. The solution is very similar to the previous problem.

The control protocol is designed as follows:

$$u_i = u_{if} + u_{ic} + u_{ia}, i = 1, \dots, n \tag{6}$$

Design a potential function Λ_{ij} :

$$\Lambda_{ij} = \begin{cases} \left(\sqrt{\tan \mu_2\left(\left|p_i - p_j\right|^2 - r_m^2\right)}\right)^2 & ,\text{ if } p_j \in \Psi_i^c \\ 0 & ,\text{otherwise} \end{cases}$$

where μ_2 is a constant.

$$\begin{aligned} \zeta_i &= -\sum_{p_j \in \Psi_i^a} \frac{\partial \Lambda_{ij}}{\partial p_i},\ i = 1, \cdots, n \\ u_{ia} &= \left(\left|u_{if}\right| + \mu_4\right) \operatorname{sgn}\left(\zeta_i\right) \end{aligned} \tag{7}$$

where μ_4 is a constant.

Theorem 2. *Using control protocols in Equations (4), (6), and (7), the system can achieve the desired formation, avoid collision and maintain communication.*

Proof. The proof is similar to Theorem 1. □

4. Simulation

The initial position of multi-agent system is arbitrary. Using control protocols in Equations (4), (6) and (7), the simulation results in Fig.3 shows changes of formation at different times, which can prove the system can achieve the desired formation, agents can avoid collision and maintain communication.

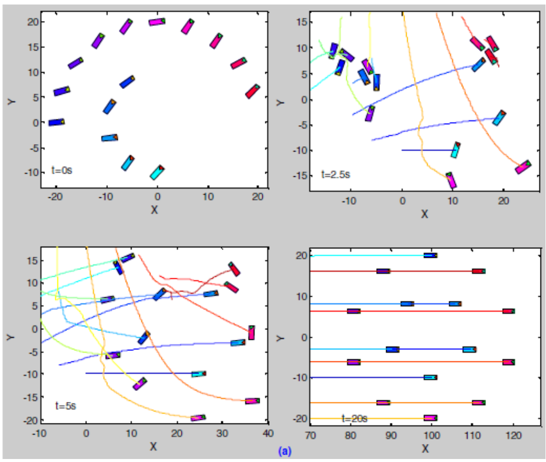

Fig. 3. Formation control for rectangular agents

4. Conclusion

For the formation control of rectangular multi-agent systems, complex Laplacian is used to form the desired formation and artificial potential function is used to maintain communication, avoid collision. For rectangular agents, global coordinate system is designed which is used to get the position of the agent and local coordinate system which is used to obtain the distance between agents. The problem of high-order nonlinear multi-agent systems can be considered in the future.

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