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Research Article

Evaluation of Two Rollers Arrangement on a Hemisphere by Kinetic Energy

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ABSTRACT

A driving roller arrangement of hemisphere is one of the important problems by omnidirectional sphere conveyance. In this research, the roller arrangement problem, viewed as an evaluation function, is thought of as mean of roller's kinetic energy with respect to the sphere direction. Furthermore, theoretically, we calculate the evaluation function, and find the contact point such that the evaluated value is minimal.

1. Introduction

A sphere, one of the basic shapes of robots, is used not only as a multi-fingered fingertip mechanism for hand robots but also as an actuator transmission mechanism for omnidirectional movement and drive in mobile robots. Spheres are also used as driving rollers for omnidirectional movement mechanisms, and there are various arrangements, depending on the application of the movement mechanism. Figure1 shows the roller contact type for the number of actuators (N_w) per sphere.

Examples of mechanisms driven by two rollers include a power transmission mechanism by Wada et al. [1] (see Figure 1(a)), a mobile device using Ishida's Figure 1(b), and The abovementioned

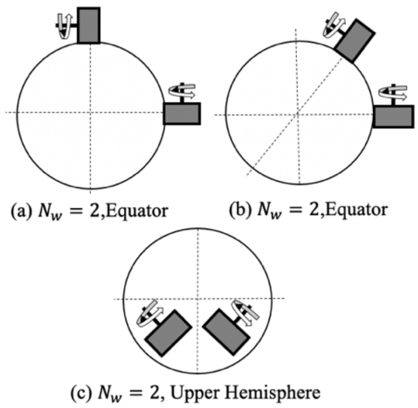

Figure 1 Type of roller arrangement for sphere mobile robot

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mechanisms can be used for the roller of a wheelchair. The rollers are arranged on the equator, generate an angular velocity vector of sphere on the horizontal plane, and can move in all direction [3]. The angular velocity vector of the sphere has two- dimensional freedom. This situation is theoretically
considered in [4].

The ball holding mechanism [5] (see Figure 1(c)) is intended to transport the ball, and the roller is placed in the upper hemisphere to hold the ball by friction.

We conducted roller arrangement problem of sphere conveyance by driving rollers as previous study[6].

In this research, in the case of omnidirectional movement, we define an evaluation function as mean of roller's kinetic energy with respect to sphere direction angle, and we also derive the exact formula. Furthermore, theoretically, we find the contact point such that the evaluated value (mean of roller's kinetic energy) is minimal. Additionally, we perform simulation and present energy distribution of several contact points on a sphere.

The remainder of this research is organized as follows. In Chapter 2, we calculate the exact evaluation value given by the integral of the kinetic energy in sphere direction. In Chapter 3, we conduct simulation on the evaluation value on the sphere. In Chapter 4, we give a summary and future issues.

2. Derivation of theoretical evaluation function

In this Chapter, we calculate the omnidirectional energy integral of the driving rollers.

As shown in Figure 2, The center $\boldsymbol{O}$ of a sphere with radius r is fixed as the origin of the coordinate system $\Sigma - xyz$. The i^{th} constraint roller (i = 1 or 2) is in point contact with the sphere at a position vector $\boldsymbol{P_i}$ ($\boldsymbol{P_1} \neq \boldsymbol{P_2}$). $\boldsymbol{\omega}$ denotes the angular velocity vector of the sphere. Because of $\boldsymbol{\eta_1}, \boldsymbol{\eta_2} \in \mathbf{span}\{\boldsymbol{P_1}, \boldsymbol{P_2}\}$ (omnidirectional condition), $\boldsymbol{\omega}$ is on $\text{span}\{\boldsymbol{P_1}, \boldsymbol{P_2}\}$. Sphere direction φ ($0° \leq \varphi < 360°$) is the angle from x-axis and ρ is the angle from xy-plane to $\boldsymbol{\omega}$. Now, given the sphere mobile velocity $\boldsymbol{V}$ (the center velocity of sphere).

2.1 Kinetic energy of the roller

Consider two rollers (right cylinder) with radius R, mass M, moment of inertia I, and roller's angular velocity ω_i
The total kinetic energy of the rollers is given by Eq. (1).

$$E = I(\omega_1^2 + \omega_2^2) \tag{1}$$

Where

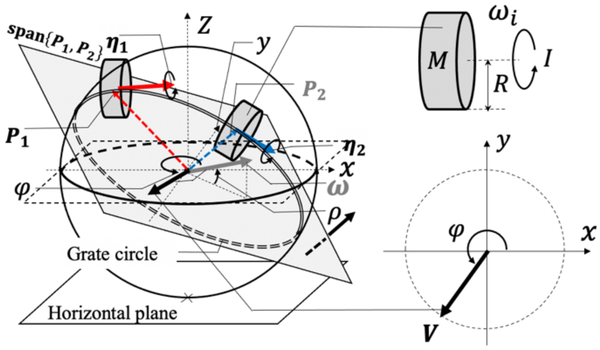

Figure 2 The sphere rotational motion by driving rollers at $\boldsymbol{P_i}$ and omnidirectional condition is $\boldsymbol{\eta_1}, \boldsymbol{\eta_2} \in \mathbf{span}\{\boldsymbol{P_1}, \boldsymbol{P_2}\}$.

$$I = \frac{1}{2} M R^2 \tag{2}$$

Because that the sphere and roller engage with each other at $\boldsymbol{P}_i$.

$$\|\boldsymbol{\omega} \times \boldsymbol{P}_i\| = R\omega_i \tag{3}$$

Thus. E is proportional with respect to sum of square two roller's speed.

$$E = \frac{M}{2}(\|\boldsymbol{\omega} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega} \times \boldsymbol{P}_2\|^2) \tag{4}$$

2.2 Mean of kinetic energy of rollers

To evaluate the value for roller arrangement, we define the follows expressions. Eq. (5) presents the mean of kinetic energy by integrating the total kinetic energy of the rollers with respect to the direction φ ($0° \leq \varphi \leq 360°$).

$$E_M = \frac{1}{2\pi}\int_0^{2\pi} E \, d\varphi \tag{5}$$

(i) Case of arbitrary roller arrangement

Quoting Equation (12) of Paper [6] (Kimura) as follows:

$$\|\boldsymbol{\omega} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\omega} \times \boldsymbol{P}_2\|^2 \tag{6}$$

$$= (\|\boldsymbol{e_3} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e_3} \times \boldsymbol{P}_2\|^2)\omega_z^2$$

$$+ 2(\langle \acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1 \rangle + \langle \acute{\boldsymbol{\omega}} \times$$

$$\boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2 \rangle)\omega_z$$

$$+ \|\acute{\boldsymbol{\omega}} \times \boldsymbol{P}_1\|^2 + \|\acute{\boldsymbol{\omega}} \times \boldsymbol{P}_2\|^2$$

where

$$\boldsymbol{P_i} = r\left[\cos\theta_{i,1}\cos\theta_{i,2}, \sin\theta_{i,1}\cos\theta_{i,2}, \sin\theta_{i,2}\right]^T \quad (7)$$

$$\boldsymbol{e_3} = [0, 0, 1]^T, \boldsymbol{\acute{\omega}} = \left[\omega_x, \omega_y, 0\right]^T \quad (8)$$

$$\omega_z = \frac{\|V\|}{r}\tan\rho \quad (9)$$

Using $\boldsymbol{P}_i = \left[P_{i,x}, P_{i,y}, P_{i,z}\right]^T$, $\boldsymbol{e_3} \times \boldsymbol{P}_i$ and $\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_i$ are represented as follow.

$$\boldsymbol{e_3} \times \boldsymbol{P}_i = \left[-P_{i,y}, P_{i,x}, 0\right]^T \quad (10)$$

$$\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_i = \left[\omega_x, \omega_y, 0\right]^T \times \left[P_{i,x}, P_{i,y}, P_{i,z}\right]^T \quad (11)$$

$$= \left[\omega_y P_{i,z}, -\omega_x P_{i,z}, \omega_x P_{i,y} - \omega_y P_{i,x}\right]^T$$

Using Eq. (10), $\|\boldsymbol{e_3} \times \boldsymbol{P}_i\|^2$ is calculated in teams of $\boldsymbol{P}_i = \left[P_{i,x}, P_{i,y}, P_{i,z}\right]^T$.

$$\|\boldsymbol{e_3} \times \boldsymbol{P}_i\|^2 = P_{i,x}^2 + P_{i,y}^2 \quad (12)$$

$$\|\boldsymbol{e_3} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{e_3} \times \boldsymbol{P}_2\|^2 \quad (13)$$

$$= P_{1,x}^2 + P_{1,y}^2 + P_{2,x}^2 + P_{2,y}^2 = 2r^2 - P_{1,z}^2 - P_{2,z}^2$$

Using Eq. (10) and Eq. (11),

$$\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_i, \boldsymbol{e_3} \times \boldsymbol{P}_i \rangle \quad (14)$$

$$= -P_{i,y}P_{i,z}\,\omega_y - P_{i,x}P_{i,z}\,\omega_x$$

$$= -\frac{\|V\|}{r}(P_{i,y}P_{i,z}\cos\varphi - P_{i,x}P_{i,z}\sin\varphi)$$

$$\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1 \rangle + \langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2 \rangle \quad (15)$$

$$= -\frac{\|V\|}{r}\{\left(P_{1,x}P_{1,z} + P_{2,x}P_{2,z}\right)\sin\varphi + (P_{1,y}P_{1,z} + P_{2,y}P_{2,z})\cos\varphi\}$$

Using Eq. (11),

$$\|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_i\|^2 = \left(\omega_x^2 + \omega_y^2\right)P_{i,z}^2 + (\omega_x P_{i,y} - \omega_y P_{i,x})^2$$

$$= (\omega_x^2 + \omega_y^2)P_{i,z}^2 + P_{i,y}^2\omega_x^2 + P_{i,x}^2\omega_y^2 - 2P_{i,x}P_{i,y}\,\omega_x\omega_y$$

$$= \frac{\|\boldsymbol{V}\|^2}{\boldsymbol{r}^2}(P_{i,z}^2 + P_{i,y}^2\sin^2\varphi + P_{i,x}^2\cos^2\varphi + 2P_{i,x}P_{i,y}\sin\varphi\cos\varphi) \quad (16)$$

$$\|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1\|^2 + \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2\|^2 = \frac{\|V\|^2}{r^2}\{P_{1,z}^2 + P_{2,z}^2 + (P_{1,y}^2 + P_{2,y}^2)\sin^2\varphi + (P_{1,x}^2 + P_{2,x}^2)\cos^2\varphi + 2(P_{1,x}P_{1,y} + P_{2,x}P_{2,y})\sin\varphi\cos\varphi\} \quad (17)$$

Thus. By substituting Eq. (13), Eq. (15) and Eq. (17) for Eq.(6), E_M can be represented in teams of $P_{i,x}$, $P_{i,y}$, $P_{i,z}$.

Here, quoting Eq. (5) and Eq. (6) in [6] (Kimura), we have

$$\tan\rho = \frac{(\boldsymbol{P_1} \times \boldsymbol{P_2})_x\sin\varphi - (\boldsymbol{P_1} \times \boldsymbol{P_2})_y\cos\varphi}{(\boldsymbol{P_1} \times \boldsymbol{P_2})_z} \quad (18)$$

$$= \frac{\left(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}\right)\sin\varphi - \left(P_{1,z}P_{2,x} - P_{1,x}P_{2,z}\right)\cos\varphi}{P_{1,x}P_{2,y} - P_{1,y}P_{2,x}}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\sin^2\varphi\,d\varphi = \int_0^{2\pi}\cos^2\varphi\,d\varphi = \frac{1}{2}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\sin\varphi\cos\varphi\,d\varphi = 0 \quad (19)$$

Using Eq. (18) and Eq. (19) ,

$$\int_0^{2\pi}\tan^2\rho\,d\varphi \quad (20)$$

$$= \frac{((\boldsymbol{P_1} \times \boldsymbol{P_2})_x)^2 + ((\boldsymbol{P_1} \times \boldsymbol{P_2})_y)^2}{((\boldsymbol{P_1} \times \boldsymbol{P_2})_z)^2}\pi$$

$$= \frac{(P_{1,y}P_{2,z} - P_{2,y}P_{1,z})^2 + (P_{1,z}P_{2,x} - P_{1,x}P_{2,z})^2}{(P_{1,x}P_{2,y} - P_{1,y}P_{2,x})^2}$$

Using Eq. (15) and Eq. (18),

$$\int_0^{2\pi}(\langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_1, \boldsymbol{e_3} \times \boldsymbol{P}_1 \rangle + \langle \boldsymbol{\acute{\omega}} \times \boldsymbol{P}_2, \boldsymbol{e_3} \times \boldsymbol{P}_2 \rangle)\omega_z d\varphi \quad (21)$$

$$= \pi\frac{\|V\|^2}{r^2}\frac{1}{P_{1,x}P_{2,y} - P_{1,y}P_{2,x}}\{(P_{1,x}P_{1,z} + P_{2,x}P_{2,z})(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}) + (P_{1,y}P_{1,z} + P_{2,y}P_{2,z})(P_{1,z}P_{2,x} - P_{1,x}P_{2,z})\}$$

Using Eq. (17),

$$\int_0^{2\pi}\|\boldsymbol{\acute{\omega}} \times \boldsymbol{P_1}\|^2 + \|\boldsymbol{\acute{\omega}} \times \boldsymbol{P_2}\|^2\,d\varphi \quad (22)$$

$$= \pi\frac{\|V\|^2}{r^2}(2P_{1,z}^2 + 2P_{2,z}^2 + P_{1,x}^2 + P_{1,y}^2 + P_{2,x}^2 + P_{2,y}^2)$$

$$= \pi\frac{\|V\|^2}{r^2}(2r^2 + P_{1,z}^2 + P_{2,z}^2)$$

Integral by $\varphi(0° \leq \varphi \leq 360°)$ is represented as follow. By substituting Eq. (13), Eq. (20), Eq. (21), and Eq. (22) into Eq. (4), E_M can be represented as

$$\frac{4r^2}{M\|V\|^2}E_M = \frac{2r^2 - P_{1,z}^2 - P_{2,z}^2}{\left(P_{1,x}P_{2,y} - P_{1,y}P_{2,x}\right)^2}\{\left(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}\right)^2$$

$$+\left(P_{1,z}P_{2,x} - P_{1,x}P_{2,z}\right)^2\} + \frac{2}{P_{1,x}P_{2,y} - P_{1,y}P_{2,x}}\{$$

$$(P_{1,x}P_{1,z} + P_{2,x}P_{2,z})(P_{1,y}P_{2,z} - P_{2,y}P_{1,z}) + (P_{1,y}P_{1,z} +$$

$$P_{2,y}P_{2,z})(P_{1,z}P_{2,x} - P_{1,x}P_{2,z})\} + 2r^2 + P_{1,z}^2 + P_{2,z}^2 \quad (23)$$

By theoretical calculation, we get the following properties.

[Property 1]: Optimality of the evaluated value
If $(\theta_{1,2}, \theta_{2,2}) = (0, 0)$ ($\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are on the equator), E_M takes the minimal value $M\|V\|^2/2$ (see Appendix(A)).

(ii) Case of symmetry roller arrangement

Especially, in case of symmetry arrangement ($P_{1,x} = -P_{2,x}, P_{1,y} = P_{2,y}, P_{1,z} = P_{2,z}$), using $(\theta_1, \theta_2) = (\theta_{1,1}, \theta_{1,2})$. Eq. (23) is represented as follow.

$$E_M(\theta_1, \theta_2) = \frac{M\|V\|^2}{4r^2}\{2r^2 - 2P_{1,z}^2 + \frac{2P_{1,z}^2(r^2 - P_{1,z}^2)}{P_{1,y}^2}\}$$

$$= \frac{M\|V\|^2}{2}\frac{(1 - \cos^2\theta_1 \cos^2\theta_2)}{\sin^2\theta_1} \quad (24)$$

$$(0° < \theta_1 < 90°, 0° \leq \theta_2 < 90°)$$

By theoretical calculation, we prove the following fact.

[Property 2]:Monotonicity of the evaluation function
(i) When θ_1 increases, $E_M(\theta_1, \theta_2)$ also decrease.
(ii) When θ_2 increases, $E_M(\theta_1, \theta_2)$ also increase.
(See Appendix(B)).

3. simulation of Evaluation value on sphere

This Chapter presents the simulation results E_M (Eq. (24)), with $0° < \theta_1 < 90°, 0° \leq \theta_2 < 90°$, $\|\boldsymbol{V}\| = 1$ [m/s], $M = 2$.

Figure 3 shows the contact points on the upper hemisphere. Table 1 shows the distribution of $E_M(\theta_1, \theta_2)$ at the contact points on the upper hemisphere in steps of θ_1 ($0° < \theta_1 < 90°$) and θ_2 ($0° \leq \theta_2 < 90°$).

As shown in Table 1, the value increases from the lower left of the table to the right and upward correspondingly (see [**Property 2**]. $E_M(\theta_1, \theta_2)$ diverges infinitely as it approaches $(\theta_1, \theta_2) = (90°, 0°)$. In particular, when $\theta_2 = 0$, $E_M(\theta_1, \theta_2)$ is constant regardless of the contact position.

As shown in [1] and [2], when two constraint rollers are placed on the equator, the evaluation value is constant regardless of the angle of the two position vectors (see [**Property 1**]).

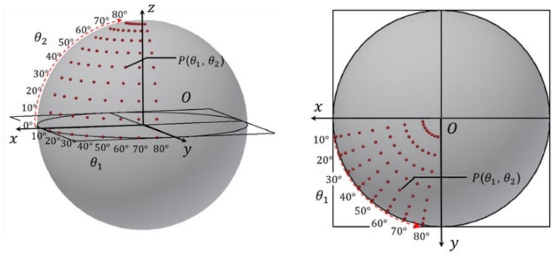

Figure 3 The distribution of contact points on the upper hemisphere. (a) Isometric view. (b) Right overhead view.

Table 1 The distribution of energy function $E_M(\theta_1, \theta_2)$ in the upper hemisphere

80°	32.19	29.40	25.12	19.87	14.29	9.04	4.76	1.97
70°	8.32	7.67	6.66	5.43	4.12	2.89	1.88	1.23
60°	3.91	3.65	3.25	2.76	2.24	1.75	1.35	1.09
50°	2.38	2.25	2.07	1.83	1.59	1.36	1.17	1.04
40°	1.68	1.62	1.53	1.41	1.29	1.18	1.08	1.02
30°	1.32	1.29	1.25	1.20	1.14	1.08	1.04	1.01
20°	1.13	1.12	1.10	1.08	1.05	1.03	1.02	1.00
10°	1.03	1.03	1.02	1.02	1.01	1.01	1.00	1.00
0°	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
θ_2,θ_1	**10°**	**20°**	**30°**	**40°**	**50°**	**60°**	**70°**	**80°**

In the ball holding mechanism (evaluation of the placement of the world team) [5], the roller arrangement is on the upper hemisphere for ball transportation, but it is less-energy efficient than on the equator. Since the ball is not fixed by a pole caster, it is required to be placed on the upper hemisphere.

4. Conclusion

In this research, we defined an evaluation function as mean of roller's kinetic energy with respect to sphere direction angle and derived the exact formula. Furthermore, theoretically, we proved that points on equator are minimal.

Future issues include consideration of motion related to variable mechanisms with offset.

Appendix

(A) Proof of [Property 1]

We replace as follow in Eq. (23).

$$P_{1,x} = p,\ P_{1,y} = q,\ P_{1,z} = x$$
$$P_{2,x} = \alpha,\ P_{2,y} = \beta,\ P_{2,z} = y \qquad \text{(A,1)}$$
$$\bar{E} = \frac{4r^2}{M\|V\|^2} E_M$$

$$\bar{E} = \frac{2r^2 - x^2 - y^2}{(p\beta - q\alpha)^2}\{(qy - \beta x)^2 + (\alpha x - py)^2\} + \frac{2}{p\beta - q\alpha}\{(px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py)\} + 2r^2 + x^2 + y^2 \qquad \text{(A,2)}$$

Where

$$p^2 + q^2 + x^2 = r^2,\quad \alpha^2 + \beta^2 + y^2 = r^2 \qquad \text{(A,3)}$$

Furthermore.

$$E_M = PX^2 + 2QX + R \qquad \text{(A,4)}$$

Where

$$X = 1/(p\beta - q\alpha),\ (p\beta - q\alpha \neq 0) \qquad \text{(A,5)}$$

$$P = (2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\} \qquad \text{(A,6)}$$

$$Q = (px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py) \qquad \text{(A,7)}$$

$$R = 2r^2 + x^2 + y^2 \qquad \text{(A,8)}$$

From $2r^2 - x^2 - y^2 > 0 \Rightarrow P \geq 0$, we consider case of $P > 0$ and $P = 0$.

(i) Case of $P > 0$

From completing the square of Eq. (A,4),

$$\bar{E} = PX^2 + 2QX + R$$
$$= P(X + \frac{Q}{P})^2 + R - \frac{Q^2}{P} \qquad \text{(A,9)}$$

$\bar{E}$ takes minimal value

$$R - \frac{Q^2}{P} = 2r^2 + x^2 + y^2 - \frac{\{(px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py)\}^2}{(2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\}} \qquad \text{(A,10)}$$

And. Equality condition is

$$PX = -Q \quad \Leftrightarrow \qquad \text{(A,11)}$$
$$\frac{(2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\}}{p\beta - q\alpha} = -(px + \alpha y)(qy - \beta x) - (qx + \beta y)(\alpha x - py)$$

We make preparations several Lemma for prove Optimality.

[**Lemma 1**]:

$$(r^2x^2 + r^2y^2 - 2x^2y^2)^2 \geq 4(r^2 - x^2)(r^2 - y^2)x^2y^2 \qquad \text{(A,12)}$$

[**Lemma 2**]:

$$r^2x^2 + r^2y^2 - 2x^2y^2 \geq 2(p\alpha + q\beta)xy \qquad \text{(A,13)}$$

PROOF:

Eq.(A,3) is substituted in right side of Eq.(A,12). Thus, it is given.

$$(r^2x^2 + r^2y^2 - 2x^2y^2)^2 \geq 4(p^2 + q^2)(\alpha^2 + \beta^2)x^2y^2 \qquad \text{(A,14)}$$

From $p\beta - q\alpha \neq 0$ ($p\beta - q\alpha = 0$ is equality condition of Cauchy-Schwarz inequality),

$$(p^2 + q^2)(\alpha^2 + \beta^2) > (p\alpha + q\beta)^2 \qquad \text{(A,15)}$$

Using Cauchy-Schwarz inequality and $x^2y^2 \geq 0$,

$$4(p^2 + q^2)(\alpha^2 + \beta^2)x^2y^2 \geq 4(p\alpha + q\beta)^2x^2y^2 \qquad \text{(A,16)}$$

Using Eq. (A,14) and Eq. (A,16), it is given.

$$4(r^2x^2 + r^2y^2 - 2x^2y^2)^2 \geq 4(p\alpha + q\beta)^2x^2y^2 \qquad \text{(A,17)}$$

Focus on $r^2x^2 + r^2y^2 - 2x^2y^2$ and AM-GM inequality.

$$r^2x^2 + r^2y^2 - 2x^2y^2 = r^2(x^2 + y^2) - 2x^2y^2$$
$$\geq r^2 \times 2\sqrt{x^2y^2} - 2x^2y^2 = 2xy(r^2 - xy) \geq 0 \qquad \text{(A,18)}$$

From $|p\alpha + q\beta| \geq 0$, $|p\alpha + q\beta| \geq p\alpha + q\beta$,

$$r^2x^2 + r^2y^2 - 2x^2y^2 \geq 2|p\alpha + q\beta|xy$$

$$\geq 2(p\alpha + q\beta)xy \quad \text{(A,19)}$$

Equality conditions are $x = y$, $x^2y^2 = 0$ and
$\{ = 0$ or $p\alpha + q\beta \geq 0 \}$.
Thus. $x = y = 0$. [END]

[**Lemma 3**]:

$$(x^2 + y^2)(2r^2 - x^2 - y^2)$$
$$\geq (px + \alpha y)^2 + (qx + \beta y)^2 \quad \text{(A,20)}$$

PROOF:
Using Eq. (A,3) and [**Lemma 2**],

$$(x^2 + y^2)(2r^2 - x^2 - y^2) - (px + \alpha y)^2$$
$$-(qx + \beta y)^2 = 2r^2(x^2 + y^2) - (x^2 + y^2)^2$$
$$-(p^2 + q^2)x^2 - (\alpha^2 + \beta^2)y^2 - 2(p\alpha + q\beta)xy$$
$$= r^2x^2 + r^2y^2 - 2x^2y^2 - 2(p\alpha + q\beta)xy \geq 0$$
(A,21)

Equality conditions are $x = y = 0$. [END]

Using [**Lemma 3**] , Cauchy-Schwarz inequality and $P > 0$,

$$(x^2 + y^2)(2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\}$$
$$\geq \{(px + \alpha y)^2 + (qx + \beta y)^2\}\{(qy - \beta x)^2 + (\alpha x - py)^2\}$$
$$\geq \{(px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py)\}^2$$
$$\Leftrightarrow \quad x^2 + y^2 \geq$$
$$\frac{\{(px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py)\}^2}{(2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\}}$$
(A,22)

From Eq. (A,10) and Eq. (A,22),

$$\bar{E} \geq R - \frac{Q^2}{P} = 2r^2 + x^2 + y^2 \quad \text{(A,23)}$$
$$-\frac{\{(px + \alpha y)(qy - \beta x) + (qx + \beta y)(\alpha x - py)\}^2}{(2r^2 - x^2 - y^2)\{(qy - \beta x)^2 + (\alpha x - py)^2\}}$$
$$\geq 2r^2$$

And. equality condition is Eq. (A,11) , $x = y = 0$ and $(px + \alpha y)(\alpha x - py) - (qx + \beta y)(qy - \beta x) = 0$.
Thus. Minimal value is $M\|V\|^2/2$ when $x = y = 0$ (The contact points are on the equator).
[END]

(ii) Case of $P = 0$

From Eq. (A,6) and $2r^2 - x^2 - y^2 > 0$,

$$qy - \beta x = \alpha x - py = 0 \quad \text{(A,24)}$$

From Eq. (A,6) and Eq. (A,24),

$$Q = 0 \quad \text{(A,25)}$$

Thus

$$\bar{E} = PX^2 - 2QX + R \quad \text{(A,26)}$$
$$= R = 2r^2 + x^2 + y^2 \geq 2r^2$$

Equality condition is $x = y = 0$.
From (i) and (ii), It is proved completely.

(B) Proof of [Property 2]

We put $X = \sin^2\theta_1$ and $Y = \sin^2\theta_2$ in Eq. (23).

$$E_M(\theta_1, \theta_2)$$
$$= \frac{M\|V\|^2}{2}\frac{1 - (1 - X)(1 - Y)}{X} \quad \text{(A,27)}$$
$$= \frac{M\|V\|^2}{2}\frac{X + Y - XY}{X}$$
$$= \frac{M\|V\|^2}{2}\left\{1 + (\frac{1}{X} - 1)Y\right\}$$

From $0 < \frac{1}{X} - 1$, $E_M(\theta_1, \theta_2)$ is an decreasing function with respect to θ_1 . and $E_M(\theta_1, \theta_2)$ is an increasing function with respect to θ_2.

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