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Research Article

Design of local linear models using PID Tuning According to error

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ABSTRACT

PID control is widely used in process systems represented by chemical processes and petroleum refining processes. The reason is that PID control has a simple structure. However, most of the existing systems are non-linear systems, and it is difficult to always obtain good control results with fixed PID control. Therefore, in this study, we propose a method of tuning the PID gain according to the deviation (control error) of the control result, and verify the effectiveness of this method through experiments. For self-tuning PID control using a local linear model, we propose a program that performs PID tuning only when the deviation occurs with a certain magnitude. A simulation is performed on the Hammerstein model, which is a non-linear system. As a result of the experiment, the number of PID gain changes could be significantly reduced.

1. Introduction

PID control [1],[2] is widely used in industries, including process systems such as chemical processes and petroleum refining processes, because it has a simple structure and its physical interpretation is clear. However, the characteristics of many existing systems vary according to environmental and operating conditions and include nonlinear systems. Therefore, it is difficult to obtain accurate control results with fixed PID control. Consequently, machine learning and data-driven [3],[4] control methods have been proposed to effectively control nonlinear systems. However, in these methods, PID tuning is performed step by step; hence, the computational processing load is high, and the processing can be performed using only a high-precision computer.
In this paper, we developed a method for tuning the PID gain only when the control result does not follow the target value. In the proposed method, a threshold is set for the deviation, and PID control is performed only when the threshold is exceeded. If the deviation is smaller than the threshold value, control is performed without changing the PID gain. We performed simulations on a bilinear model, which is a nonlinear system, and verified the effectiveness of this method when focusing on the reduction of computational load and control results. We propose a method for tuning the PID gain according to the deviation (control error) of the control result and verify its effectiveness based on numerical examples.

2. A design of a self-tuning control system using a local linear model that performs PID tuning according to deviation

Fig. 1 shows a block diagram of the proposed control system. The authors have previously proposed a method for calculating control parameters using the concept of the local linear model [5]. This method can control a nonlinear system by locally establishing a linear model.

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In the proposed control method, the deviation is evaluated for a self-tuning control system using a local linear model. The nonlinear system is controlled by performing PID tuning only when a deviation occurs to a certain magnitude.

2.1. *System description*

First, consider the discrete-time nonlinear system represented by

$$y(t) = f(\varphi(t-1)) \tag{1}$$

where, y(t) represents the system output and f(·) represents the nonlinear function. Also, φ(t-1) represents the state of the system before time t-1 (historical data) and is called the information vector. The information vector φ(t-1) is defined by the following equation.

$$\varphi(t-1) := [y(t-1),\ y(t-2), \cdots, y(t-n_y)\ \ u(t-1), u(t-2), \cdots, u(t-n_u)\,] \tag{2}$$

Furthermore, u(t) is the control input, and n_y and n_u are the orders of the output and input, respectively. Now, suppose that the nonlinear system represented by equation (1) can be locally represented by a linear model as follows

$$A_i(z^{-1})y(t) = z^{-(k_m+1)}B_i(z^{-1})u(t) \quad (i = 1,2,\cdots,N) \tag{3}$$

where, in Eq. (3), km represents the minimum estimate of the lag time, and when the lag time is known, km is set to that value; when the range of the lag time is unknown, k_m is set to 0. Furthermore, z^{-1} represents a time-delay operator, meaning $z^{-1}y(t) = y(t-1)$. Also, $A(z^{-1})$ and $B(z^{-1})$ are given by

$$A_i(z^{-1}) = 1 + a_{i,1}z^{-1} + \cdots + a_{i,n_y}z^{-n_y} \tag{4}$$

$$B_i(z^{-1}) = b_0 + b_{i,1}z^{-1} + \cdots + b_{i,n_u}z^{-n_u} \tag{5}$$

After the above preparation, the controller is designed for the local linear model.

2.2. *Controller Design*

Design the controller based on the following steps.

[STEP1] Construction of multiple linear models

For the nonlinear model, multiple linear models are constructed, system identification is performed using the lumped least squares method, and the parameters of $A_i(z-1)$ and $B_i(z-1)$ (i = 1, 2, - - - , N; where i takes these values unless otherwise noted) included in the linear model of Eq. (3) are estimated.) parameters in the linear model.

[STEP2] Design of control system

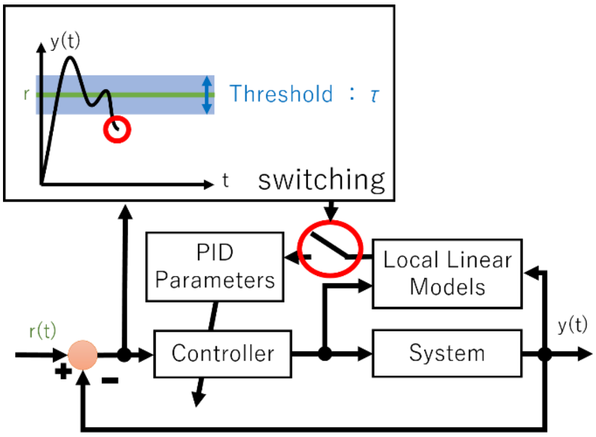

Fig. 1 Block diagram

For the linear model represented by Eq. (3), consider the feedback control law given by Eq.

$$R(z^{-1})y(t) + S(z^{-1})\Delta u(t) - R(1)r(t) = 0 \tag{6}$$

where, r(t) represents the target value at step t. $R(z^{-1})$ and $S(z^{-1})$are polynomials designed based on the pole configuration of the closed-loop system, respectively.

$$\left.\begin{aligned} R_i(z^{-1}) &= r_{i,0} + r_{i,1}z^{-1} + \cdots + r_{i,n_1}z^{-n_1} \\ S_i(z^{-1}) &= 1 + s_{i,1}z^{-1} + \cdots + s_{i,n_2}z^{-n_2} \end{aligned}\right\} \tag{7}$$

$R(z^{-1})$ and $S(z^{-1})$ are designed based on the pole placement method. In this case, the input-output relation of the closed-loop system composed of equations (3) and (6) is expressed by the following Eq.

$$y_i(t) = \frac{z^{-(k_m+1)}B_i(z^{-1})R_i(1)}{P(z^{-1})}r(t)$$

(8)

The denominator polynomial $P(z^{-1})$ of Eq.(8) is defined by the following Eq.

$$P(z^{-1}) := \Delta A_i(z^{-1})S_i(z^{-1}) + z^{-(k_m+1)}B_i(z^{-1})R_i(z^{-1}) \tag{9}$$

It can be seen that $P(z^{-1})$ is the characteristic polynomial of the closed-loop system. The following equation is used to design this polynomial.

$$P(z^{-1}) = 1 + p_1z^{-1} + p_2z^{-2} \tag{10}$$

$$\left.\begin{aligned} P_1 &= -2e^{\frac{\rho}{2\mu}}\cos\left(\frac{\sqrt{4\mu-1}}{2\mu}\right) \\ P_2 &= e^{-\frac{\rho}{\mu}} \\ \rho &:= \frac{T_s}{\sigma} \\ \mu &:= 0.25(1-\delta) + 0.51\delta \end{aligned}\right\} \tag{11}$$

σ indicates a parameter corresponding to the rise time, and μ is a parameter related to the damping characteristics of the response, which is adjusted by δ.
Here, $R(z^{-1})$ and $S(z^{-1})$ are calculated based on Eq.(9). In order to obtain $R(z^{-1})$ and $S(z^{-1})$ uniquely, it is necessary to set their orders to $n_1 = n_y$ and $n_2 = n_u + k_m$, respectively. In this way, the pole placement control system can be designed for each linear model.
[STEP3] Replacement with PID controller
We have described a control method based on the pole placement method. This method can be replaced by a design method based on PID control if it is considered in the same way as in Eq.(6). First, consider the PID control law of the following Eq.

$$\Delta u(t) = K_I e(t) - K_P \Delta y(t) - K_D \Delta^{-2} y(t) \quad (12)$$

where, K_P , K_I , and K_D represent the PID gain, respectively. Furthermore, e(t) is the control deviation, which is given by the following Eq.

$$e(t) := r(t) - y(t) \quad (13)$$

Now, Eq.(6) is rewritten as follows

$$\frac{R_i(z^{-1})}{\nu} y(t) + \Delta u(t) - \frac{R_i(1)}{\nu} r(t) = 0 \quad (14)$$

In this case, from equations (12) and (14), the PID parameter is given by

$$\left.\begin{aligned} K_P &= \frac{-r_{i,1} - 2r_{i,2}}{\nu} \\ K_I &= \frac{r_{i,0} + r_{i,1} + r_{i,2}}{\nu} \\ K_D &= \frac{r_{i,2}}{\nu} \end{aligned}\right\} \quad (15)$$

The above allows us to adjust the PID parameters based on the approximate pole configuration.

$$\nu := 1 + s_{i,1} + s_{i,2} \quad (16)$$

[STEP4] Calculation of weights
Next, for each local linear data calculated in [STEP2], the estimation error $\varepsilon_i(t)$ is calculated for each model, and the weight ω_i is calculated based on this. $\varepsilon_i(t)$ is the error between the system output value y(t) and the estimated output value $\hat{y}(t)$ of each linear model. Here, $\hat{y}(t)$ is calculated based on equation (3) by the following formula

$$\hat{y}_i(t) = -A_i(z^{-1}) y(t) + z^{-(k_m+1)} B_i(z^{-1}) u(t) \quad (17)$$

$$\varepsilon(t) = \left| y(t) - \hat{y}_i(t) \right| \quad (18)$$

where, A_i (z^(-1)) and B_i (z^(-1)) are the system parameters of each linear model estimated in [STEP1].

$$\omega_i(t) = \frac{\frac{1}{\varepsilon_i(t)}}{\sum_{i=1}^{N} \frac{1}{\varepsilon_i(t)}} \quad (19)$$

In addition, $\omega_i(t)$ is the weight corresponding to the selected i-th information vector. The smaller the difference between the actual outputs value of the system and each linear model, the larger the value of this weight. Note that the calculation of $\omega_i(t)$ based on equation (18) satisfies the following Eq.

$$\sum_{i=1}^{N} \omega_i(t) = 1 \quad (20)$$

[STEP5] Generation of weighted PID parameters
Using the weights obtained in [STEP4] and the PID parameters in Eq.(15), calculate the weighted PID parameters using the following Eq.

$$\mathbf{K}(N) = \sum_{i=1}^{N} \omega_i(t) \mathbf{K}(i) \quad (21)$$

$$\mathbf{K}(i) := \left[K_P(i), K_I(i), K_D(i) \right] \quad (22)$$

3. PID tuning method in response to deviation

We explain how to perform PID tuning only when the deviation reaches a particular magnitude with respect to the competitive tuning PID control using the local linear model in Section 2.2. First, the threshold is defined as τ. The condition for τ is expressed by the following equation.
.

$$\tau > |e(t)| \quad (23)$$

The PID control described in Section 2.2 is performed only when this condition is satisfied, and the control is performed without changing the PID gain if it is not satisfied. Here, the parameter τ included in Eq. (23) denotes the design parameter given by a certain positive constant. Setting this parameter requires trial and error.

4. Simulation

In order to verify the effectiveness of this method, a numerical example for a nonlinear system is presented. The Bilinear model is used as the control target. The Bilinear model, which is expressed by the following equation, is considered as the control target.

$$y(t) = 0.4y(t-1) - 0.99y(t-2) + 0.3u(t-1)$$

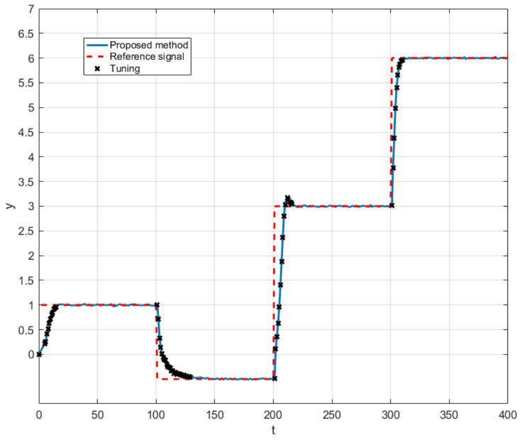

Fig. 2 Control result using the newly proposed PID control scheme

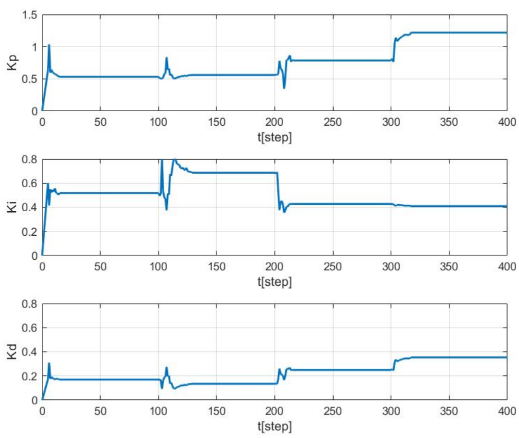

Fig.3 Temporal change in PID of the proposed method

$$-0.1u(t-2)+0.1y(t-1)u(t-1) \\ +0.05y(t-2)u(t-2)+\xi(t) \qquad (24)$$

However, ζ(t) is a Gaussian white noise with mean 0 and variance 0.012. The target value is given as follows.

$$r(t)=\begin{cases}1.0(0\le t<100)\\ -1.0(100\le t<200)\\ 3(200\le t<300)\\ 6(300\le t\le 400)\end{cases} \qquad (25)$$

Next, based on the static characteristics, a linear model corresponding to the control input range is constructed as follows. However, the number of divisions is set to N=3. The various design parameters included in this method are $n_y=2$, $n_u=1$, and $k_m=0$.

$$\begin{cases}-4.0\le u_1<2.0\\ 2.0\le u_2<3.4\\ 3.4\le u_3<4.0\end{cases} \qquad (26)$$

Furthermore, the threshold τ for determining PID tuning is set to 0.1. Also, the number of experiments is 400. Fig.2 shows the control results obtained using the proposed method. Fig.3 shows the temporal change in PID gain. The points shown in Fig. 2 indicate that the PID gain is tuned. Based on the results in Fig. 2 and Fig. 3, the control results of the conventional data-driven type and the proposed method are compared, and it is confirmed that the difference is slight. However, the number of changes in the PID gain needs to be varied 400 times because the conventional method tunes the PID gain sequentially. Using the proposed method, the number of changes was 56, which was approximately 1/7 of that of the conventional method; this is a significant decrease in the number of PID gain changes. This result suggests that the computational cost can be significantly reduced and can be implemented on a low-function computer.

5. Conclusions

In this paper, we present a self-tuning control system using a local linear model that performs PID tuning according to the deviation for a nonlinear system. Specifically, we propose a method that defines a threshold value and changes the PID gain to control only when the conditions related to the threshold value are satisfied. In addition, a simulation was performed on the bilinear model, a nonlinear system, to verify the effectiveness of the proposed method. In the future, we plan to evaluate the control performance of the deviation threshold setting using the item response theory.

References

1. K.Heong Ang, G. Chong, Y. Li,"PID control system analysis, design, and technology", IEEE Transactions on Control Systems Technology, Vol.13, Issue.4, 2005
2. C. Gao, F. Wang, Z. Fu,"Fuzzy self-tuning PID control algorithm for belt conveyor driven by multi-motor", Journal of Robotics, Networking and Artificial Life, pp.175-178, 2017
3. T.Yamamoto, K.Takao and T.Yamada: Design of a Data-Driven PID Controller, IEEE Trans. on Control Systems Technology, Vol.17, No.1, pp.29-39, 2009

4. T. Kinoshita, T. Yamamoto,"Design of a Data-Driven Controller with Evaluating Controller Performance", Journal of Robotics, Networking and Artificial Life, pp.257-260,2019
5. S. Imai and T. Yamamoto, "Design of a Self-Tuning Controller using Multiple Local Linear Models for Nonlinear Systems", The transactions of the Institute of Electrical Engineers of Japan. C, A publication of Electronics, Vol.132,No.6, pp.860-865, 2012 (In Japanese)

Authors Introduction

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