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Research Article

A Perfect Play on 4×12 Board of Othello

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ABSTRACT

In 1993, Joel Feinstein discovered one of perfect plays for Othello (Reversi) on a board with 2n×2n (n = 2,3) , which showed that the second player(White) is the winner, respectively. Since 2015, we have been analyzing Othello on a board with 4×2n (n = 3,4,…) . As results, we found one of perfect plays for each on the 4×2n (n = 3,4,5) boards, which showed that the first player (Black) is the winner respectively. Recently, we completed the analysis of the 4×12 board. In this paper, we will introduce the analysis results of 4×12 board Othello and summarize the results that have been found so far.

1. Introduction

Othello [1] is a board game derived from Reversi by Goro Hasegawa (JPN) in 1973. Its rules are completely unified , whereas Reversi has many local rules. The rules of Othello in this paper are the same as those of Hasegawa’s Othello, except for the size of the board.

Othello (Reversi) is one of the games with perfect information and no element of chance. In a two-player game with perfect information like Othello, they are always categorized as either a win, a loss, or a draw for the first player [2].

In 1993, Joel Feinstein found one of perfect plays for Othello with board size of 2n×2n (n = 2,3) in [3], which showed that the second move player (White) is the winner, respectively. Here, "perfect play" is the behavior or strategy of a player that leads to the best possible outcome for that player regardless of the response by the opponent.

We have been analyzing Othello with a board size of 4×2n (n = 3,4,…) since 2015. As results, we found one of perfect plays for each on the 4×2n (n = 3,4,5) boards, which showed that the first player (Black) is the winner [4], [5], [6], respectively.

Based on the results obtained from analyzing Othello so far, it seems that the second player has the advantage on a square board, while the first player has the advantage on a rectangular board with differing lengths and widths. However, this conclusion may be inaccurate. According to computer-generated high-quality games analysis of board size of 2n×2n (n=2,3,4,…), the first player's advantage increases as n increases. Therefore, it is believed that the first player always wins for any sufficiently large n. The standard 8×8 board Othello is still remains unsolved, but it is believed that it is likely to end in a draw. It may be that there are only two sizes of

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Othello board on which the first player wins: the 4×4 board and the 6×6 board.

After solving the 4×2n (n=3,4,5) board of Othello, we challenged the 4×12 board problem by partitioning it into several sub-problems. This is because its search space of the 4×12 board was too large (estimated to be more than 1000 times larger than the 4×10 boards). Recently, we solved the 4×12 board problem by solving all its sub-problems, which showed that the first player (Black) is the winner. In this paper, we introduce the analysis results of 4×12 board Othello and summarize the results that have been found so far.

2. Rules of Othello*

The rules of Othello [1] are as follows. Othello always begins with the setup as shown in Fig. 1. One player uses the black side of the pieces (circular chips), the other the white side. The player who chooses the black stone is the first to move.

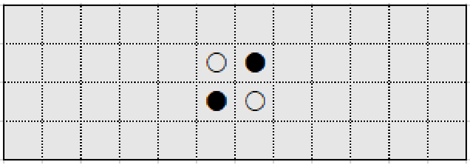

Fig. 1. Starting position for the 4×12 board.

Each player puts a piece of own color to an empty square in own turn. A player's move consists of outflanking the pieces of his opponent, he flips outflanked pieces to their own color. To outflank means to place the piece on the board so that his opponent's rows of the piece are bordered at each end by the piece of his color. If a player cannot make any move, then he has to pass. If he is able to make a valid move, then passing is not allowed. The game ends when neither player can make any valid move. The winner is the player who has more pieces than his opponent.

3. Techniques

In the combinatorial game such as Chess, Go and Othello, game tree is a directed graph whose nodes are positions in a game and whose edges are moves. The complete game tree for a game is the game tree from starting position and containing all possible moves from each position. Analyzing the complete game tree leads us to solve the game. i.e. find a sequence of moves that either the first or second player can follow that will guarantee the best possible outcome for that player. Depending on evaluation values used, we can find a perfect play.

Perfect analysis of Othello is useful to refer to the thinking routines of a game program. This is because end-game routine of game program is the perfect analysis, and evaluation function in the middle-game routine is available for the ordering of the search in perfect analysis.

3.1. *MiniMax with Alpha-Beta Pruning*

As the method of analysis in this research, regardless of the middle or the end, MiniMax with Alpha-Beta pruning [4], [5], [6] is used. MiniMax algorithm achieves the best moves for both players, Alpha-Beta method cuts unnecessary search in MiniMax. These are commonly used together since MiniMax with Alpha-Beta pruning gives the same results as the simple MiniMax search. Here, for the 4×2n (n = 2,3,4,5,6) board Othello, the evaluation value of the leaf vertex on the game tree, the final phase of game, is the stone difference between the first player and the second player. Additionally, only in the case of 4×12 board Othello, if one of the players wins completely without leaving any stones of the opponent, the winner is considered to have filled all the squares with his own stones.

3.2. *Move Ordering*

The appropriate order in the search process facilitates pruning by Alpha-Beta method. In our program, we extract all positions one step forward from the current position and sort them. In order to realize efficient search results, it is necessary to have a good evaluation value for the position. This is because the higher the accuracy of the position evaluation value, the higher the search space reduction effect. To determine the evaluation value of the position, we utilize pre-calculated values obtained through techniques such as machine learning [6].

4. Preliminary results [4], [5], [6]

This chapter introduces one of the perfect plays obtained so far on both 4×2n (n = 2,3,4,5) and 6×6 boards. However, a perfect play in 4×2n (n = 4,5) is used in the sense of the first player wins completely (the second player has no stones). The result on the 4×12 board has the same meaning.

4.1. *4×4 and 6×6 boards*

On 4×4 and 6×6 square boards, the second player wins, respectively. For the 4×4 board, a perfect play is shown in Fig. 2 which means Black: 3, White: 11. There was no pass in this procedure.

2	3	4	7
1	○	●	
6	●	○	
9	8	10	5

Fig. 2. A perfect play of 4×4 board Othello.

For the 6×6 board, a perfect play is shown in Fig. 3 which means Black: 16, White: 20. It was passed at move 31.

28	9	8	7	32	33
22	14	1	4	17	16
27	13	○	●	6	15
19	2	●	○	5	12
24	23	3	10	26	29
21	18	20	11	25	30

Fig. 3. A perfect play of 6×6 board Othello.

There are generally multiple perfect plays. Fig. 4 shows Feinstein's perfect play which has no pass.

23	9	8	7	27	25
24	14	1	4	17	16
20	13	○	●	6	15
19	2	●	○	5	12
21	30	3	10	31	29
22	18	26	11	28	32

Fig. 4. Feinstein's perfect play.

Feinstein's perfect play is different from that of Fig. 3, but the score is the same.

4.2. *4×6, 4×8 and 4×10 boards*

On 4×6, 4×8, and 4×10 rectangular boards, the first player wins, respectively. For the 4×6 board, a perfect play is shown in Fig. 5 which means Black: 20, White: 4. It was passed at move 18.

3	2	1	10	6	11
16	5	○	●	8	19
17	14	●	○	9	10
15	21	4	13	7	12

Fig. 5. A perfect play of 4×6 board Othello.

For the 4×8 board, a perfect play is shown in Fig. 6 which means the first player wins completely (the second player has no stones remaining), with Black: 28, White: 0. It was passed at move 12, 24 and 26, respectively.

15	3	2	1	4	5	19	
20	14	10	○	●	8	22	27
25	17	13	●	○			
21	16	9	6	11	7	18	23

Fig. 6. A perfect play of 4×8 board Othello.

For 4×10 board, a perfect play is shown in Fig.7 which means the first player wins completely (the second player has no stones remaining), with Black: 39, White: 0. It was passed at move 19, 28, 34, 36, 38 and 40, respectively.

21	20	3	2	1	4	5	13	30	31
24	17	16	14	○	●	10	22	29	32
25	26	23	15	●	○	18	39	35	33
27	37	8	7	6	11	9	12	41	

Fig. 7. A perfect play of 4×10 board Othello.

The evaluation value in Othello on a 4×2n (n=4,5) board is the stone difference between the first player and the second player. Perfect play in this paper means the best play in this sense. For this reason, for example, in Othello on a 4×10 board, the first player may always win before 39 moves. However, at that time, the stone difference will be less than 39 stones.

4.3. *4×12 board*

The first 6 moves in each perfect play of 4×2n (n = 3,4,6) boards introduced in the previous section are identical. Therefore, we examined the perfect play of the 4×12 board with the initial position fixed up to the 6th move, using the same method as for the 4×2n (n = 3,4,5) board, see Fig. 8. Our findings indicate that the first player achieves a complete victory, leaving the second player with no remaining stones [6].

29	19	18	❸	②	❶	④	❺	13	32	41	
30	24	17	16	14	○	●	10	20	31		
35	36	21	23	15	●	○	22	33	34	39	
37	25	26	8	7	⑥	11	9	12	27		

Fig. 8. A perfect play of 4×12 board Othello with the initial position fixed up to the 6th move.

5. Experiments

In order to find a perfect play on the 4×12 board, we examine all possible moves for White (the second player) up to the 6th move of white's turn in the procedure of Fig. 8.

Fig. 9, Fig. 10 and Fig. 11 represent the positions that must be examined in order to prove that gives win for Black (the first player) on the 4×12 board. The 'F' in each figure represents the location where White (the second player) placed their stone in the corresponding turn (6th move, 4th move, 2nd move) as shown in Fig. 8. And the Greek alphabets in Fig. 9, Fig. 10 and Fig. 11 represent all the possible positions where White can place his stone instead of F in that White's turn.

			●	●	●	●	●				
					○	●	(β)				
				(α)	●	○					
					F						

Fig. 9. 6th move.

			●	●	●	F					
					○	●	(ε)				
				(γ)	●	○					
					(δ)						

Fig. 10. 4th move.

				F	●	(η)					
					●	●					
				(ζ)	●	○					

Fig. 11. 2th move.

Under the fixed setting up to the 6th move and searching from that position, it is necessary to solve 95 sub-problems in order to examine all the alternative moves for White. However, under the assumption that this problem is a complete win for Black, we only need to search for promising moves in Black's turn and demonstrate that all result in a complete win for Black. Also, as a result, we find a perfect play.

Table 1 shows all possible 14 sub-problems, which arise from restricting the moves on Black's turn to only the promising ones after choosing alternate moves indicated by Greek alphabet for each White's move (6th move, 4th move, 2nd move). Here, the coordinates are as shown in Fig.12.

Table 1. Sub-problems.

6th move	4th move	2th move
(α)	(γ)f4g1	(ζ)f4e1d1g1
(β)	(δ)e2g1	(ζ)f4e1d1h2
	(δ)e2h1	(ζ)f4g1h1h2
	(ε)h3f4	(η)h1e1d1e3
	(ε)h3h4	(η)h1e1d1f4
		(η)h1e1d1h2
		(η)h1e3g4h3

We have confirmed that the 14 sub-problems listed in Table 1 lead to complete victories for Black (the first player). In other words, 4×12 board Othello is a game that the first player wins if both players do their best.

a1	b1	c1	d1	e1	f1	g1	h1	i1	j1	k1	l1
a2	b2	c2	d2	e2	○	●	h2	i2	j2	k2	l2
a3	b3	c3	d3	e3	●	○	h3	i3	j3	k3	l3
a4	b4	c4	d4	e4	f4	g4	h4	i4	j4	k4	l3

Fig. 12. Coordinates on 4×12 board.

For 4×12 board, a perfect play is shown in Fig. 13 indicating that the first player achieves a complete victory (the second player has no stones, that is, Black: 42, White: 0). It was passed at move 28, 30 and 40, respectively.

29	19	18	3	2	1	4	5	13	32	41	
30	24	17	16	14	○	●	10	20	31		
35	36	21	23	15	●	○	22	33	34	39	
37	25	26	8	7	6	11	9	12	27		

Fig. 13. A perfect play of 4×12 board Othello.

Our conclusion is that 4×12 board Othello, the first player can achieve a complete victory regardless of the choices made by the second player. A complete victory for the first player in 4×12 board Othello means that winning without leaving any stones for the second player, resulting in a score of 48 for Black and 0 for White. For this reason, there is also another perfect play for 4×12 board, as shown in Fig. 14, where the first player achieves a complete victory (the second player has no stones, that is, Black: 37, White: 0). It was passed at move 10, 18, 20, respectively. For this reason, for example, in Othello on a 4×12 board, the first player can always achieve a complete victory within 37 moves.

	17	16	3	2	1	4	5	23	31		
		37	13	21	○	●	22	30	29	32	33
		14	8	6	●	○	11	12	28	34	35
		25	7	19	9	24	15	26	27		

Fig. 14. Another perfect play of 4×12 board Othello.

6. Conclusions

The Game of Othello on a 4×2n (n = 3,4,5,6) board result in a victory for Black (the first player). Especially for n = 4,5,6, it is a complete victory for Black. It is highly probable that for any Othello game played on a 4×2n (where n ≧ 7) board, it always results in a complete victory for Black.

It is important to note that the strategy for achieving a complete victory with the minimum number of moves remains unresolved, as the focus has primarily been on determining the outcome of the game rather than optimizing the number of moves required for victory.

For Othello played on a 2n×2n (n = 2,3,4,…) square board, it has been established that White (the second player) can only win when n=2,3. However, the outcome remains unresolved for n greater than or equal to 4. As n increases, Black generally holds an advantage, so it is uncertain whether White is guaranteed to win for n greater than or equal to 4.

From the above, it appears that among the various sizes of Othello boards, the 4×4 and 6×6 boards are the exceptions where White can win.

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