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Journal of Robotics, Networking and Artificial Life
Vol. 9(4); March (2023), pp. 374–378
ISSN (Online):2352-6386; ISSN (Print):2405-9021
https://alife-robotics.org/jrnal.html

Research Article

A Distributed Optimal Formation Control for Multi-UAV System

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ARTICLE INFO

Article History
Received 22 November 2021
Accepted 23 October 2022

Keywords
Multi-agent system
UAV
Optimal control
Distributed formation control

ABSTRACT

In this paper, the distributed optimization problem for multi-agent system (MAS) formation control of unmanned aerial vehicles (UAVs) is suggested here. The situation that the internal states of a single UAV can be made fully available was aimed at, the internal optimal control law of a single UAV is designed using the optimal control theory. To cope with the obstacle that each agent in the system can only communicate with its neighbor agents, the distributed formation control law of the system is introduced based on the communication topology of the system, and further the stability of the system is analyzed in helps of graph theory. The validity of the suggested scheme is verified by both the numerical simulation and UAV platform.

1. Introduction

As the capabilities of robots continue to be improved, the application fields of robots are also expanding at the same time. However, just like humans, a single robot will show lower abilities in many cases, and in this way, the coordination of multiple agents is required to play a more significant role. The ability of distributed coordination and cooperation between agents is the foundation of the multi-agent system (MAS).

The problem of multi-agent coordinated control includes consensus control [1], rendezvous control [2], formation control [3], etc. Besides, there is optimization control [4] based on the above-mentioned coordinated control. Optimal control is an important bridge from theory to engineering. However, the optimization of a MAS has a strong dependence on the system communication network. Because the optimal control law needs to be able to obtain all the states of the collective individuals in the system in real-time, otherwise the conditions of optimal control cannot be met. To deal with this problem, we designed a distributed optimization formation control protocol, which is not influenced by the system topology. And this result is applied to the formation control of multiple unmanned aerial vehicles (UAVs), which is shown in Fig.1.

The main contribution of this paper mainly includes three aspects: 1) A formation control protocol based on static consensus protocol is designed. 2) The optimal control law of a single agent is studied while the performance function is optimal. 3) Combining the formation control protocol and the optimal control law, a distributed optimization formation control protocol is designed. The protocol combined the distributed advantage of formation control and the optimization advantage of optimal control. The system is able to perform formation tasks even if some agents cannot communicate with each other.

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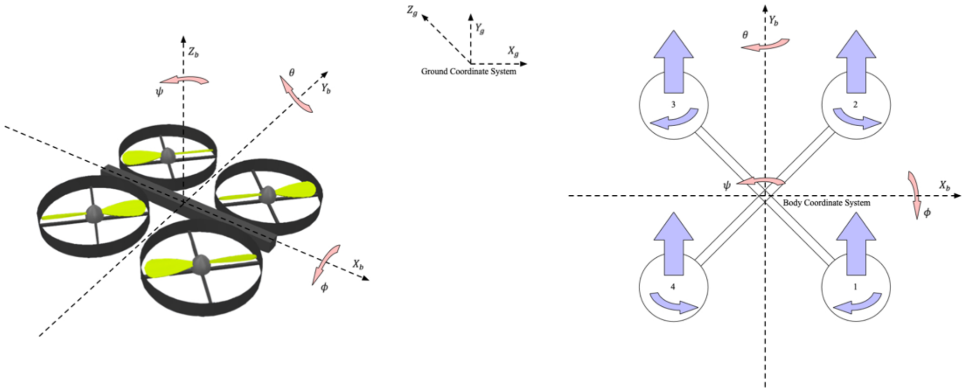


Fig.1. The illustration of UAV kinematics

2. Preliminaries

This section introduces the preliminary knowledge for studying UAV systems, including the use of graph theory to describe the communication relationships within the system, the dynamic model of a single UAV, and the state-space equation of the UAV system.

2.1. Graph theory

In order to describe the relationship of UAV systems, graph theory is used [1]. $\mathcal{A} = [a_{ij} \in \{0,1\}]$ is the adjacency matrix of the graph $\mathcal{G}$, indicating whether there is information exchange from node j to i or not. The matrix $\mathcal{D}$ is the in-degree matrix. The neighbors of node i are N_i. The Laplacian matrix is defined as

$$L = \mathcal{D} - \mathcal{A} \tag{1}$$

2.2. UAV model description

As shown in Fig.1, the UAV model can be simplified as equation (2) [5].

$$\begin{aligned} \ddot{x} &= g\theta \\ \ddot{y} &= -g\phi \\ \ddot{z} &= u_h/m - g \\ \ddot{\phi} &= u_\phi/I_x \\ \ddot{\theta} &= u_\theta/I_y \\ \ddot{\psi} &= u_\phi/I_z \end{aligned} \tag{2}$$

where x, y, z are positions in the ground coordinate system along X_g, Y_g, Z_g, respectively, ϕ, θ, ψ are roll angle, pitch angle, yaw angle of the UAV, respectively. I_x, I_y, I_z are inertia moments along X_b, Y_b, Z_b in the body coordinate system, respectively. And u_h is the lift from four propellers.

For the sake of easy computation, system (2) is transformed into the state-space format, listed in equation (3).

$$\dot{X} = AX + BU \tag{3}$$

where $X = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad g\theta \quad -g\phi \quad 0 \quad g\dot{\theta} \quad -g\dot{\phi} \quad 0]^{\mathrm{T}}$, $U = [u_\phi \quad u_\theta \quad 0]^{\mathrm{T}}$,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes I_3, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \otimes I_3,$$

$\otimes$ is the Kronecker product.

When there have N UAVs, the state-space equation (3) can transform into the following form.

$$\dot{\tilde{X}} = I_N \otimes A \tilde{X} + I_N \otimes B \tilde{U} \tag{4}$$

where $\tilde{X} = [X_1^{\mathrm{T}}, X_2^{\mathrm{T}}, \ldots, X_N^{\mathrm{T}}]^{\mathrm{T}}, \tilde{U} = [U_1^{\mathrm{T}}, U_2^{\mathrm{T}}, \ldots, U_N^{\mathrm{T}}]^{\mathrm{T}}$, I_N represent an N-dimensional identity matrix.

The desired formation state h is defined as follows

$$h = \begin{bmatrix} h^x \\ h^y \\ h^z \end{bmatrix} \in \mathbb{R}^3 \tag{5}$$

Take the formation states (5) into the position states, three new error position states come naturally as (6).

$$\begin{bmatrix} \delta^x \\ \delta^y \\ \delta^z \end{bmatrix} = \begin{bmatrix} x - h^x \\ y - h^y \\ z - h^z \end{bmatrix} \tag{6}$$

Then the formation control problem is turned into finding a protocol $\tilde{U}$ to drive the error vector δ to zero, which indicates that

$$\lim_{t\to\infty} \|\delta_i - \delta_j\| = 0, \quad \lim_{t\to\infty} \|v_i\| = 0,$$
$$\lim_{t\to\infty} \|\Omega_i\| = 0, \quad \lim_{t\to\infty} \|\dot{\Omega}_i\| = 0, \quad i = 1,2,\cdots,N. \tag{7}$$

3. Main results

This section starts with the design of the formation control protocol. After that, the optimal control law of a single UAV is provided in detail. Finally, the optimal control law is extended to the formation control protocol in collection.

Lemma 1 [6]**.** For a $N \times N$ Laplacian matrix L, $\mathrm{e}^{-Lt}, \forall t \geq 0$ is a random matrix with positive diagonal elements. If L has a unique zero eigenvalue, $\mathrm{Rank}(L) = N-1$, then its left eigenvector has $v = [v_1 \quad v_2 \quad \cdots \quad v_N]^{\mathrm{T}} \geq 0$ and $1_N^{\mathrm{T}} v = 1, L^{\mathrm{T}} v = 0$, where $t \to \infty, \mathrm{e}^{-Lt} \to 1_N v^{\mathrm{T}}$.

3.1. Formation control

Referring to the previous work, the consensus protocols can be divided into two types: One is static and the other one is dynamic. Based on the static consensus protocol, the following formation control protocol is employed here as (8)

$$u_i = \alpha \sum_{j \in N_i} (\delta_j - \delta_i) - \beta v_i - \gamma_1 \Omega_i - \gamma_2 \dot{\Omega}_i \quad (8)$$

where α, β, γ_1 and γ_2 all of them are positive gains. $\delta_i = [\delta_i^x, \delta_i^y, \delta_i^z]^{\mathrm{T}}, v_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^{\mathrm{T}}$, $\Omega_i = [g\theta_i, -g\phi_i, 0]^{\mathrm{T}}, \dot{\Omega}_i = [g\dot{\theta}_i, -g\dot{\phi}_i, 0]^{\mathrm{T}}$.

Theorem 1. Assume G is the connected undirected graph. The system (4) can realize the formation as defined in (7) if protocol (8) satisfies the following conditions:

$$\alpha > 0, \beta > 0, \gamma_1 > 0, \gamma_2 > 0, \beta \gg \alpha,$$
$$\gamma_1, \gamma_2 > \beta \gg \alpha, \beta\gamma_1\gamma_2 > \beta^2 + \gamma_2^2 \alpha$$

Proof. Please referring to Reference [5]. □

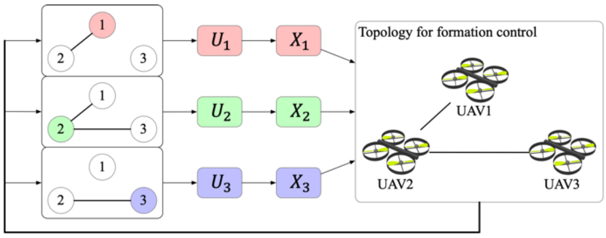
3.2. Optimal control

The formation control just requires the communication topology is strongly connected, as shown in Fig.2(a). As shown in Fig.2(b), the optimal control needs each agent to be able to communicate with all agents in the system, which is often not satisfied in the multi-agent system. Fortunately, the state of each agent itself is fully available. Therefore, the optimal control law in a single agent can be studied to determine the optimal control law of the system.

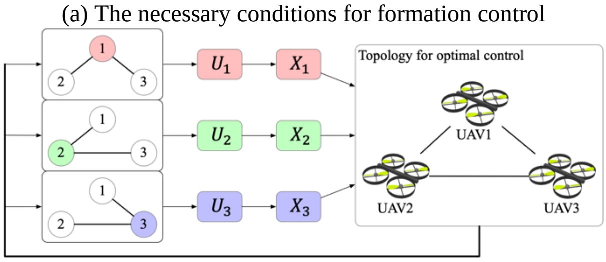
Before giving the performance function, let $\bar{X} = [\delta^x \quad \delta^y \quad \delta^z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad g\theta \quad -g\phi \quad 0 \quad g\dot{\theta} \quad -g\dot{\phi} \quad 0]^{\mathrm{T}}$, then the performance function is defined as

$$J_i = \int_0^\infty \left[\bar{X}_i^{\mathrm{T}}(t) Q \bar{X}_i(t) + u_i^{\mathrm{T}}(t) R u_i(t)\right] dt \quad (9)$$

Since the states of UAV on different coordinate axes are independent, we need to set weight matrices $Q = q * I_{12}, R = r * I_3$, where $q > 0, r > 0$.



(a) The necessary conditions for formation control



(b) The necessary conditions for optimal control

Fig.2. System communication topology conditions

Through the optimal control theory, the optimal control law of single agent is

$$u_i^* = -R^{-1} B^T P \bar{X} \quad (10)$$

where P is the solution of the Riccati equation (11).

$$A^{\mathrm{T}} P + PA - PBR^{-1} B^{\mathrm{T}} P + Q = 0 \quad (11)$$

3.3. Distributed optimization formation control

Through the above calculation, the optimal control law (10) can be obtained. Let $K = R^{-1} B^T P$, the dimension of K must be 3×12, and it also has the following form $K = [k_1 \quad k_2 \quad k_3 \quad k_4] \otimes I_3 =$

$$\begin{bmatrix} k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 & 0 \\ 0 & 0 & k_1 & 0 & 0 & k_2 & 0 & 0 & k_3 & 0 & 0 & k_4 \end{bmatrix}^{\mathrm{T}}$$

The multi-agent system of UAV is isomorphic, that is, the dynamic performance of all agents is the same, so the optimal control law can be directly applied to the UAV system. Therefore, the optimal formation control law is obtained as follows

$$u_i = k_1 \sum_{j \in N_i} (\delta_j - \delta_i) - k_2 v_i - k_3 \Omega_i - k_4 \dot{\Omega}_i \quad (12)$$

where k_1, k_2, k_3, k_4 come from matrix K.

Theorem 2. Assume G is connected undirected graph. The UAV system (4) can complete the formation while making the performance function (9) optimal if it uses the protocol (12).

Proof. Please referring to Reference [7]. □

4. Simulations

In this section, numerical simulation and virtual platform simulation were carried out to verify the effectiveness of the designed formation control protocol.

4.1. Experiment 1

The numerical experiments have verified the formation control and distributed optimization formation control respectively, as shown in Fig.3 and Fig.4.

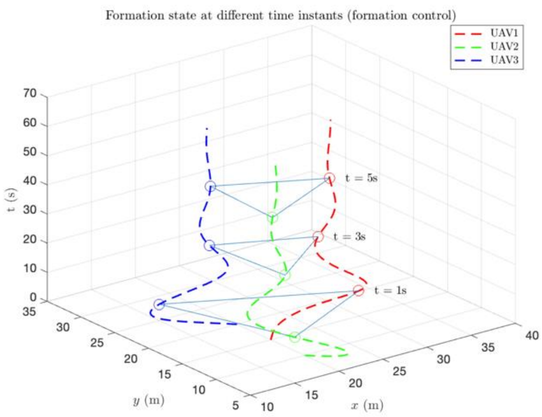


Fig.3. System states formation based on formation control

In the formation control, it can be observed that the system can complete the triangular formation, but there is a large error from the set position during the completion of the formation, and there is still an error at the 5th second. When using distributed optimal formation control, it can be observed that the system can quickly complete the triangular formation, and the error between the actual position and the set value is small.

In addition, it is surprising that the optimal control not only reduces the loss of the system but also accelerates the convergence speed of the system, which is of great help in reducing the time it takes for the system to form a formation.

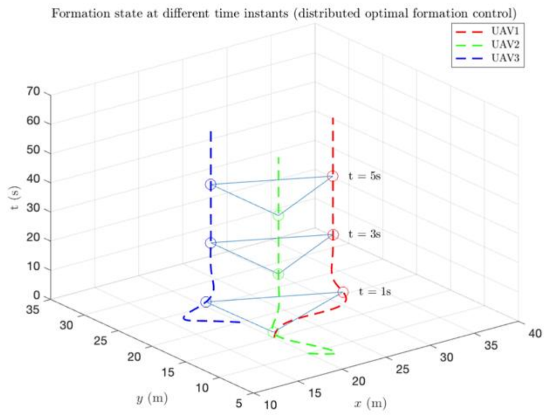
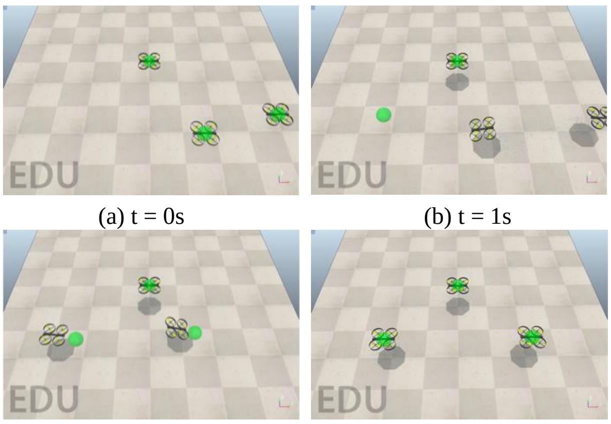


Fig.4. System states based on distributed optimization formation control

4.2. Experiment 2

The simulation software CoppeliaSim/Vrep was also used to verify the flight of the UAV formation. To keep the system in view during the operation, we assume that a UAV remains stationary. Observing the position of the system at different time instants, as shown in Fig.5, it can be seen that the system can finally complete the triangular formation. The full version of the experiment video can be seen at

https://www.bilibili.com/video/BV1Br4y1D7VJ/.



(a) t = 0s (b) t = 1s

(c) t = 3s (d) t = 5s

Fig.5. The position of three UAVs at different time instants

In this paper, the protocol is only verified in a simulated environment. The biggest challenge between the simulation environment and the real environment is that the real environment has many kinds of disturbances. Our next step is to improve the robustness and interference immunity of the protocol.

5. Conclusions

In this paper, a multi-UAV system is established by analyzing the dynamic model of UAV. Based on the static consensus protocol, the formation control protocol is designed. An optimal control law in a single UAV is designed for solving the problem that UAV in the system cannot obtain the status information of all other UAVs. A distributed optimization formation control protocol for multi-UAV system is designed is designed by combining the formation control protocol and the optimal control law. The protocol is not interfered with by the communication topology of the multi-agent system. Finally, the coeffective of the protocol is verified by the numerical simulation and virtual platform.

In the future, we plan to study the optimal control of heterogeneous system.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China (No. 61340041,61374079 and 61903126) and the Natural Science Foundation of Henan Province (182300410112).

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Authors Introduction

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