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Research Article

Wheeled Mobile Robot Robust Control Based on Hybrid Controller Approach

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ABSTRACT

In this paper, a novel trajectory following method based on hybrid control strategy is proposed for the trajectory following problem of mobile robots with nonholonomic constraints. The mobile robot controller is composed of three parts: kinematics, dynamics equations and Cerebellar Model Articulation Controller. The control speed needed to realize the reference trajectory tracking is obtained from the motion equation under the position tracking error. The virtual control value of the control speed is obtained by using Lyapunov, and the controller is designed in the dynamic equation. The cerebellar model articulation controller (CMAC) is used to approximate the nonlinearity and uncertainty of the dynamic model of the mobile robot. At the same time, the torque controller is combined with the velocity error to form the torque controller. The influence of the uncertain disturbance on the system is compensated on-line. In the discussion in this paper, the nonlinear system is consider, and the Lyapunov stability criterion guarantees the global stability of the system and the asymptotical convergence of tracking error. The simulation results in Matlab / Simulink environment further verify the effectiveness and superiority of the proposed control algorithm.

1. Introduction

Many literatures which study nonholonomic mobile robots are usually focus on wheeled mobile robots (WMR). In the theoretical research of WMR's motion control, in general, only pure rolling condition is considered. In other words, it is assumed that no slip condition (including lateral and longitudinal sliding) occurs. This ideal constraint is essentially a kind of nonholonomic constraint, therefore, WMR is a typical case of the nonholonomic system. In this paper, the control of WMR is studied. According to different control objectives, the control problems of nonholonomic systems can be divided into three categories [1],[2],[3]: position stabilization, trajectory tracking and path following.

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Path following control only follows a designed path regardless of the arriving time of the specific position. According to some control theory, a control law is designed for the nonholonomic mobile robot system, so that the nonholonomic mobile robot can reach and eventually follow a given path on the motion plane at a given speed. Here, a given path is also called the expected path, which refers to a geometric curve, $f(x_r, y_r, \theta_r)$, the curve equation does not contain time parameters, and each argument is not a function of time, Where x_r is the reference coordinate x, $y_r$is the reference coordinate y, and θ_r is the WRM body direction, which is represented by the reference coordinates (x, y).

The given velocity refers to the given linear velocity v_r and angular velocity ω_r, also known as the expected speed. The reference control input expressed in $u_r = [v_r, \omega_r]^T$. From the physical point of view, the value change of v_r and ω_r is limited by the specific form of

$f(x_r, y_r, \theta_r)$, and also can not include the time parameter, and $v_r > 0$ is required. u_r and $f(x_r, y_r, \theta_r)$ can be given in advance or generated by path generator. Hence, we can regard the path following problem as a special case of the trajectory tracking problem since the former is much easier to be deal with compared to the later.

The organization of this paper is in the following. Section 1, we introduction and briefly discuss the basic concepts of path following control. The path following control of nonholonomic WMR are summarized the research motivation and main structure of this paper are illustrated in the Section 2 paragraph. In Section 2, the control path following of nonholonomic WMR is summarized, and the research motivation and main structure of this paper are described. It also describes these controllers constructed from kinematics and dynamics equations. The cerebellar model articulation controller algorithm similar to neural network is used to design online compensation system and path error to overcome the uncertainty of WRM, which will be described in Section 3. compensation system for uncertainties and the controller. The cerebellar neural network weight updating algorithm and kernel space algorithm are introduced in Section 3. The architecture of the WRM robust hybrid controller in Section 4. Section 5 given the numerical simulation results of WRM and conclusions.

2. Kinematic Model and Controller Design

2.1 Model of WMR

The dynamics equations of a nonholonomic mobile robot with *n*-dimensional state space and subject to m constraints read [4],[5],[6],[7]

$$M(q)\ddot{q}(t) + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \boldsymbol{\tau}_d = B(q)\boldsymbol{\tau} - A^T(q)\lambda \quad (1)$$

$$\dot{q} = S(q)u \quad (2)$$

where $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric, positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the centripetal and Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravitational vector, and $F(\dot{q}) \in \mathbb{R}^n$ is the surface friction. To a mobile robot moving on a smooth plane, vectors $G(q)$ and $F(\dot{q})$ are equal to zero. $\tau_d \in \mathbb{R}^n$ denotes bounded unknown disturbances, $\boldsymbol{\tau} \in \mathbb{R}^p$ denotes the control input, $B(q) \in \mathbb{R}^{n\times p}$ denotes the input transformation matrix, $\lambda \in \mathbb{R}^m$ is the constraint force vector, and $A(q) \in \mathbb{R}^{m\times n}$ is the constraint matrix.

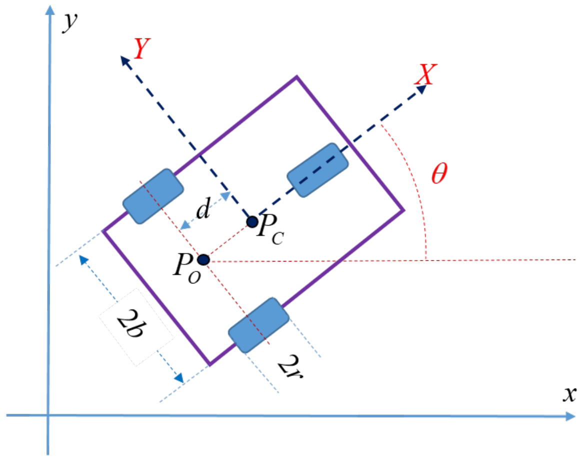

Figure 1 WMR nonholonomic mobile robot.

Figure 1 illustrates a three wheels mobile robot, in which d is the distance between robot's mass center P_C, P_O denotes the geometric center, $2b$ is the distance between two driving wheels, and r denotes the wheel radius. $q = [x \quad y \quad \theta]^T$ represents robot's position and orientation, $u = [v \quad \omega]^T$ represents velocity and angular velocity, $\boldsymbol{\tau} = [\tau_1 \quad \tau_2]^T$ is control torque, J denotes the inertia moment, and m_a denotes the mass of the WMR. The parameters in each matrix appeared in Eqs. (1) and (2) are

$$M(q) = \begin{bmatrix} m_a & 0 & m_a d \sin\theta \\ 0 & m_a & -m_a d \cos\theta \\ m_a d \sin\theta & -m_a d \cos\theta & J \end{bmatrix}, C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & m_a d\dot{\theta}\cos\theta \\ 0 & 0 & m_a d\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}, B(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ b & -b \end{bmatrix}, A^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix}, S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}$$

We can express nonholonomic constraints as

$$A(q)\dot{q} = 0 \quad (3)$$

Set a *n-m* dimensional full rank matrix $S(q)$ as a base set in null space $A(q)$ such that

$$A(q)S^T(q) = 0. \quad (4)$$

We can obtain an auxiliary velocity control input $u \in \mathbb{R}^{n-m}$ from Eqs. (2) and (4),

$$\dot{q} = S(q)u. \quad (5)$$

From Eq. (5) we can have

$$\ddot{q} = S(q)\dot{u} + S(\dot{q})u. \quad (6)$$

By inserting Eq. (6) into Eq. (1) and multiplying by $S^T(q)$ to cancel the constraint matrix $A^T\lambda$, we obtain the dynamic equation of the nonholonomic mobile robot:

$$S^T MS\dot{u} + S^T\left(M\dot{S} + CS\right)u + S^T\boldsymbol{\tau}_d = S^T B\boldsymbol{\tau} \quad (7)$$

After variable replacements, Eq. (7) becomes

$$\bar{M}(q)\dot{u} + \bar{C}(q,\dot{q})u + \bar{\boldsymbol{\tau}}_d = \bar{\boldsymbol{\tau}} \quad (8)$$

where $\bar{M}(q) \in \mathbb{R}^{p\times p}$ is the symmetric positive definite inertia matrix, $\bar{C}(q,\dot{q}) \in \mathbb{R}^{p\times p}$ is matrix for centripetal and Coriolis forces, $S^T\tau_d = \bar{\boldsymbol{\tau}}_d \in \mathbb{R}^p$ contains unstructured unmodeled dynamic bounded unknown perturbations, and $S^T B\tau = \bar{\boldsymbol{\tau}} \in \mathbb{R}^p$ is control input matrix ; in which $\bar{M} = \begin{bmatrix} m_a & 0 \\ 0 & J - m_a d^2 \end{bmatrix}$, $\bar{C} = 0_{2\times 2}$, $\bar{\boldsymbol{\tau}}_d = S^T(q)\boldsymbol{\tau}_d$, and $\bar{\tau} = S^T(q)B(q)$.

2.2 *Kinematic Controller Design*

$q_r = [x_r \quad y_r \quad \theta_r]^T$ is the reference orientation of the WMR

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -d\sin\theta_r \\ \sin\theta_r & d\cos\theta_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \quad (9)$$

where x_r, y_r, θ_r, v_r and ω_r are the expectations of x, y, θ, v and ω.

A. Kinematics Controller Design

Let us define tracking error as

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} = T_e \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (10)$$

where $T_e = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (11)

With the differentiation of Eq. (10), we can have the attitude error with respect to time expressed as:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_r\cos\theta_e \\ v_r\sin\theta_e \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & y_e \\ 0 & -x_e \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (12)$$

From the above analysis, we can define the trajectory tracking of the mobile robot kinematics model as finding the bounded input $v$and ω such that for any initial error, system (12) is bounded by the control input $(x_e, y_e, \theta_e)^T$ and satisfies the condition $\lim_{t\to\infty}\|(x_e, y_e, \theta_e)^T\| = 0$.

Lemma 1: To any $\varphi \in \mathbb{R}$ and $\|\varphi\| \in \mathbb{R}$, the condition $f(\varphi) = \varphi\, sin(arc\,tan(\varphi)) \geq 0$ is true, if and only if $\varphi = 0$.

B. $\bar{\tau}_d$ Disturbance Observer Design

There are two main factors affecting the robustness of the WMR system. One is the interference from the outside world; while the other is the uncertainty of the internal parameters of the whole system. In fact, the WMR will be affected by friction and various noise such that there are some differences compared to the ideal system. The disturbance observer is designed to estimate the external disturbance, $\bar{\tau}_d$. The external disturbance estimator designed for the WMR system is usually in the form of [3],[6],[7]:

$$\begin{cases} \hat{\bar{\tau}}_d = z + Lu \\ \dot{z} = L\bar{M}^{-1}z - L(-\bar{M}^{-1}Lu - \bar{M}^{-1}\bar{C}u + \bar{M}^{-1}\bar{\tau}) \end{cases} \quad (13)$$

where $\hat{\bar{\tau}}_d$ denotes the estimation of the unknown external disturbance $\bar{\tau}_d$, z is the internal state of the nonlinear estimator, L is the parameter of the nonlinear estimator needed to be solved, which is usually expressed in terms of the constant matrix.

Some assumptions are needed to control the WMR to track the target trajectory. Let us state these assumptions in the following.

Assumption 1: The disturbances $\|\bar{\boldsymbol{\tau}}_d\|$ and $\|\dot{\bar{\boldsymbol{\tau}}}_d\|$ are bounded, and $\bar{\boldsymbol{\tau}}_d$ is constant in the steady state, i.e., $\lim_{t\to\infty}\|\dot{\bar{\boldsymbol{\tau}}}_d\| = 0$.

Assumption 2: The first derivative of the reference linear velocity v_r and angular velocity ω_r are bounded and $v_r > 0$.

Lemma 2: $L\bar{M}^{-1}$ is the Hurwitz matrix, and $\tilde{\bar{\tau}}_d$ is asymptote convergence, then we express $\bar{\tau}_d = \hat{\bar{\tau}}_d$.

C. The Output Layer Setting of the Cerebellar Model Controller

Consider Eqs. (12) and (14), and introduce a new control variable, $\bar{u} = u_c - u$

$$\begin{aligned} \bar{M}\dot{\bar{u}} &= -\bar{C}(q,\dot{q})\bar{u} + (\bar{M}\dot{u}_c + \bar{C}(q,\dot{q})u_c) + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \\ &= -\bar{C}(q,\dot{q})\bar{u} + \Gamma(u_c,\dot{u}_c) + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \end{aligned} \quad (14)$$

where $\Gamma(u_c,\dot{u}_c) = \bar{M}\dot{u}_c + \bar{C}(q,\dot{q})u_c$ is the nonlinear function of the mobile robot, function $\Gamma(u_c,\dot{u}_c)$ includes many parameters of the mobile robot such as mass, rotation inertia and so on. It is very difficult to determine these parameters, hence we use cerebellar model controller to approach, which can be expressed as

$$\Gamma(u_c, \dot{u}_c) = Y(u_c, \dot{u}_c)W \tag{15}$$

where $Y(u_c, \dot{u}_c)$ denotes the cerebellar network output, and W is the connection weighting of the cerebellar network output. Then we can rewrite Eq. (14) as

$$\bar{M}\dot{\bar{u}} = -\bar{C}\bar{u} + YW + \bar{\boldsymbol{\tau}}_d - \bar{\boldsymbol{\tau}} \tag{16}$$

3. Cerebellar Model Articulation Controller

Cerebellar Model Articulation Controller (CMAC) is defined by a series of mapping [8][9]：

CMAC mapping： $\boldsymbol{P} \rightarrow \boldsymbol{I_v} \rightarrow \boldsymbol{H_b} \rightarrow \boldsymbol{Y}$

where $\boldsymbol{P}$ is the input vector, $\boldsymbol{I_v}$ is a set of internal variables, $\boldsymbol{H_b}$ is a set of hypercube, and $\boldsymbol{Y}$ is the output vector.

We choose δ learning rule to adjust the weight,

$$E = \frac{1}{2c}e^2(t) \tag{17}$$

where $e(t) = r(t) - y(t)$ is the error, c is generalization parameter shown in Figure 2.

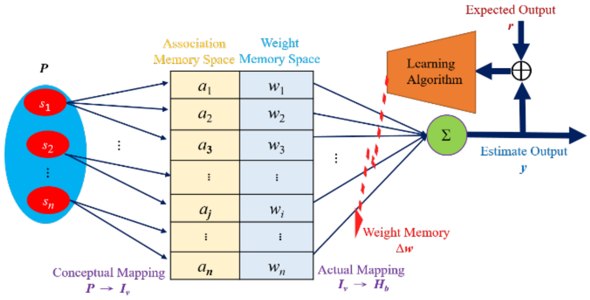

Figure 2 Scheme of a CMAC model

And adopt gradient decent method,

$$\Delta w_j(t) = -\eta\frac{\partial E}{\partial w} = \eta\frac{e(t)}{c} \tag{18}$$

$$w_j(t) = w_j(t-1) + \Delta w_j(t) + \alpha\left(w_j(t-1) - w_j(t-2)\right) \tag{19}$$

where η is the learning speed, and α is inertial coefficient.

4. The Architecture of The WRM Robust Hybrid Controller

The scheme of WMR control used in this paper is illustrated in Figure 4. By giving virtual control input to provide virtual linear velocity and angular velocity according to WMR's kinematic model. The actual linear velocity and angular velocity are given by torque controller , which enables the WMR to track the virtual linear velocity and angular velocity. Then, the external disturbances and internal uncertainties act on the torque output are compensated by the CMAC neural network shown in Figure 3.

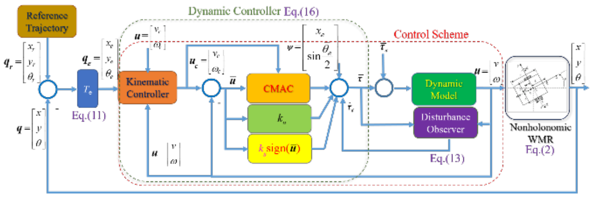

Figure 3 Scheme of CMAC neural network control WMR output torque to compensate the external disturbance

5. Simulation Results and Conclusions

In order to verify the actual control performance provided by the proposed algorithm, we use MATLAB / SIMULINK to execute the simulation. Parameters of the WMR are list as follows: the wheel radius, $r = 0.12\,m$, the distance between two driving wheels, $2b = 0.6\,m$, the mass of the WMR, $m_a = 4\,kg$, the distance between robot's mass center and geometric center, $d = 0.25\,m$and the inertial moment, $J = 0.25\,kgm^2$. In the simulation, the parameters of the observer and controller are designed as $L = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}$, where $L_1 = L_2 = 12m$, and $k_x = k_y = k_\theta = 1$, $k_u = 0.3$ and $k_s = 0.8$. The initial position of the reference input is $[x_r(0) \quad y_r(0) \quad \theta_r(0)] = [0 \quad 0 \quad 0]$. The reference velocities are set as $v_r = 0.1\,m/s, \omega_r = 0.01\,rad/s$. The initial state of the WMR is given as $[x(0) \quad y(0) \quad \theta(0)] = [39.5m \quad 0.1m \quad 89.427deg.[]]$ .The external disturbance $\bar{\tau} = \left[1.2\,sin(0.3t - \frac{\pi}{2}) \quad 0.51\,sin(0.3t + \frac{\pi}{2}) \quad -1.31\,sin(0.5t)\right]^T$ occurs after the WMR is moving. Learning rate of CMAC controller is $\eta = 0.5$, and the inertia coefficient is $\alpha = 0.5$.

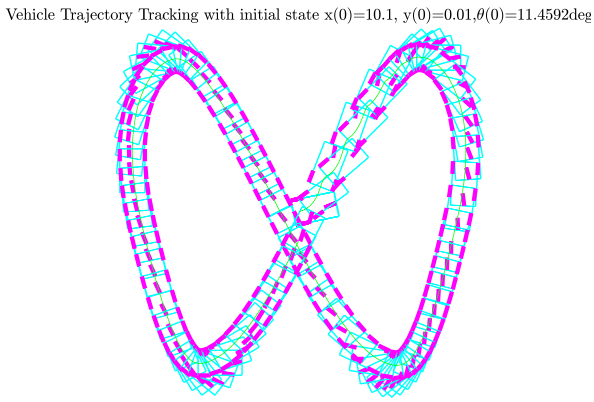

Figure 4 WRM trajectory tracking

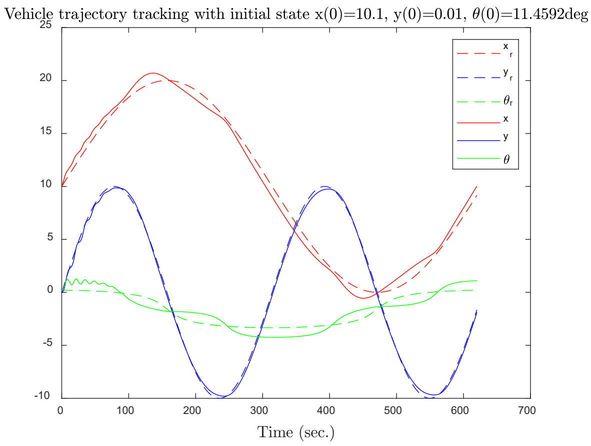

Figure 5 WRM trajectory following expected and real path

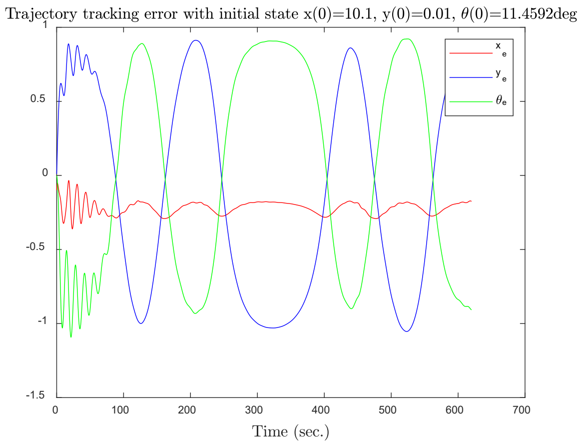

Figure 6 WRM trajectory following path error

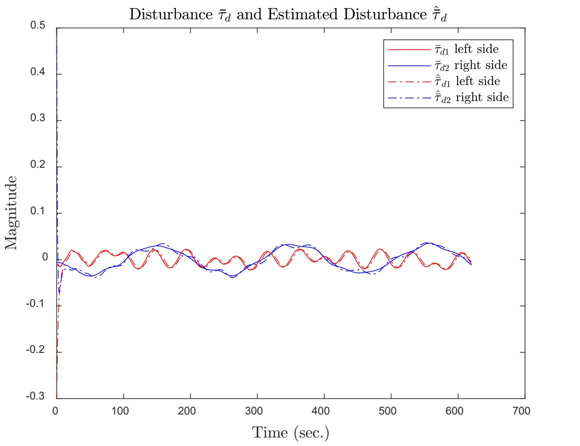

Figure 7 WRM disturbance$\bar{\tau}_d$and estimated disturbance $\hat{\bar{\tau}}_d$

Figure 4. 5, 6, 7 show WRM trajectory tacking, expected and real path, following path error, disturbance and estimated disturbance.

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Authors introduction

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He is an Associate Professor of Department of Aviation & Communication Electronics at Air Force Institute of Technology in Taiwan. He received his D. Eng. degree in Electrical Engineering, both from National Cheng Kung University, Taiwan, in 2002. His research interest is micro-satellite attitude control.