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Research Article

Research on characteristics and synchronization control of a system without equilibrium point

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ABSTRACT

In this paper, the dynamic characteristics of a chaotic system without equilibrium point are studied. Through numerical simulation and theoretical analysis, the chaos characteristics of the system without equilibrium point are studied. In this paper, the synchronous control method based on state observer is used. First, the state observer system is designed according to the original system. And then the simulation results are obtained. At the end of this paper, we give the advantages and disadvantages of this synchronization method.

1. Introduction

Chaos is a kind of motion which is more complex and special than other forms in nonlinear dynamic system. It can be found everywhere in nature. Known as the three great revolutions of physics in the 20th century, relativity and quantum mechanics are the other. With the development of the research on chaos system, the emergence of the system without equilibrium point and the analysis of its characteristics are becoming more and more popular [1].

Chaos synchronization usually means that there are at least two vibration systems and their phases can be coordinated. Although chaos synchronization and chaos control are independent in definition, it is generally considered as a special form of chaos control in academia. Before the chaos synchronization is proposed, the traditional chaos control is to stabilize the system on the track of unstable period, and the chaos synchronization is to reconstruct the two systems. Many researchers put forward many concepts of synchronization, such as complete synchronization, same step, generalized synchronization, lag synchronization and so on [2]. In addition to the rich research results in theory, there are also many synchronization schemes in experiments, such as Guan Xin equal based on neural network, studied the synchronization problem of two chaotic systems in the presence of disturbance, Lu Xiang and so on designed a delay synchronization controller to realize the synchronization control between spatiotemporal chaotic Systems [3]. At present, scholars are keen to combine electronic circuit, laser system, neural network and computer system with chaos synchronization, so as to realize some application schemes of chaos control and synchronization. Therefore, chaos synchronization is one of the hottest research directions in the current chaos, and also one of the most widely used topics. Its future prospects are very broad.

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2. Theoretical basis of chaotic system

Chaos can be seen everywhere in our life, such as the reproduction of biology, the fluctuation of stock market, the sudden change of geomagnetic field and so on. Chaos is a kind of seemingly random but actually deterministic motion. Compared with the mechanical motion in classical mechanics, it can predict the motion state of the system at a certain time in the future through calculation and analysis according to the system state at the initial time and the deterministic law. In chaotic system, the state of the system will not be accurately predicted, and only the future motion trend can be determined.

Li Tianyan, an American Chinese scholar, and his thesis tutor York first proposed the concept of chaos, and gave the mathematical definition of chaos.

Li Yorke definition [4].

The mapping f contains all periodic points. There is an uncountable subset $S \in I$ without periodic points, and the following conditions are satisfied:

① $\forall x, y \in S, x \neq y \lim_{n \to \infty} sup \left| f^{(n)}(x) - f^{(n)}(y) \right| > 0$

② $\forall x, y \in S, \lim_{n \to 0} inf \left| f^{(n)}(x) - f^{(n)}(y) \right| = 0$

③ $\forall x \in S, \lim_{n \to 0} sup \left| f^{(n)}(x) - f^{(n)}(y) \right| > 0$

f which satisfies the above conditions, is said to be chaotic on set s. From the mathematical definition put forward by two scholars, we can get the essential characteristics of a chaotic system, which are boundedness, aperiodicity and initial value sensitivity.

In 1989, there was a new definition of Devaney [5]. Let X be a metric space. A continuous mapping: $f: X \rightarrow X$ is called chaos on X, and f satisfies the following conditions:

(1) f is topologically transitive;

(2) The periodic points of f are dense in X

(3) f is highly sensitive to the initial conditions.

The most essential characteristic of chaotic dynamic system is the sensitivity of initial value, which is the famous butterfly effect. This concept refers to two identical systems, and their initial values have extremely small differences. After a period of change, the output values of the two systems will be greatly different, just like two completely different systems.

2.1. Research methods of chaos

In order to understand the chaotic nature of the system, we usually simulate the bifurcation diagram, time sequence diagram, phase trajectory diagram and Lyapunov exponent spectrum of the system to analyze the system.

Select the time parameter as the ordinate and take a separate state quantity as the abscissa. The graph drawn in this way is called the sequence diagram. In this way, we can draw all the state variables into one graph, so we can get the phase trace graph of the system. From the time sequence diagram, we can clearly know whether the current state of the system is in chaos or periodic state through numerical changes. From the phase trajectory, we can clearly and accurately judge the state of the system from the shape of the attractor.

In a general nonlinear system, we assume that there is a parameter variable, when the parameter variable changes very little near a specific value, then the small change will cause a huge change in the topological properties of the phase space. This phenomenon is called bifurcation, and the critical point is defined as the bifurcation value, when the parameter is on the coordinate axis In the abscissa of, we call the point of bifurcation and define the point that does not cause bifurcation as the constant point.

As a common phenomenon, bifurcation is very important in flight linear system. In order to get the bifurcation diagram, we need to get all the values of the parameters in a certain range, so that the topological properties of the trajectory lines in the phase space will be shown, and the properties shown in the diagram are the bifurcation diagram we want. When the parameters are taken to different values, the system will also show different states. After analysis, we can see that the occurrence of bifurcation points means that the system will be unstable, so the bifurcation points are sometimes called unstable points.

Lyapunov index is a very important reference. It is an index law to measure the attraction or divergence of adjacent orbits in phase space, and judge whether the adjacent orbits in phase space attract or diverge according to its positive and negative energy. When one of the Lyapunov exponents of the system is less than zero, it means that the motion of the system in this direction will be a stable state [6], otherwise, when one of the Lyapunov exponents of the system is greater than zero, it means that the motion of the system in this direction is unstable, or the phase space of the system will be an expanded state, that is, the adjacent trajectory will change more and more Divergence, because of this nature, we can see that the long-term behavior of the system in this state is unpredictable. Therefore, it can be realized to judge the state of the system according to its Lyapunov index.

Here we take a three-dimensional chaotic system as an example to illustrate the relationship between the Lyapunov exponent and the state of the system [7]. Because the system is a three-dimensional system, it is not difficult to see that the system has three Lyapunov exponents. The three of them together are called the Lyapunov exponent spectrum. After calculation, when all of them are negative, the attractor is called fixed point and the system is in steady state. When one of them is zero and the other two are negative, the attractor is periodic and the system is in periodic state. The difference between stationary state and periodic state is not obvious, so they can also be regarded as the same state. If one of them is zero and the other two are not zero, and the attractor is a chaotic attractor, then the system is in a chaotic state [8].

2.2. hematical model of three-dimensional chaotic system without equilibrium point

The state equations of the three-dimensional chaotic system studied in this paper are as follows：

$$\begin{cases} \dot{x} = ay + xz \\ \dot{y} = -bx + yz \\ \dot{z} = 1 - x^2 - y^2 \end{cases}$$

Among them, $x, y, z \in R$ is the state variable of the system, and the parameters a = 0.05, b = 1 are selected. The phase trajectory of the system and the projection of chaotic attractor on each phase plane are simulated by MATLAB software. Phase trace diagram is shown in the Fig.1.

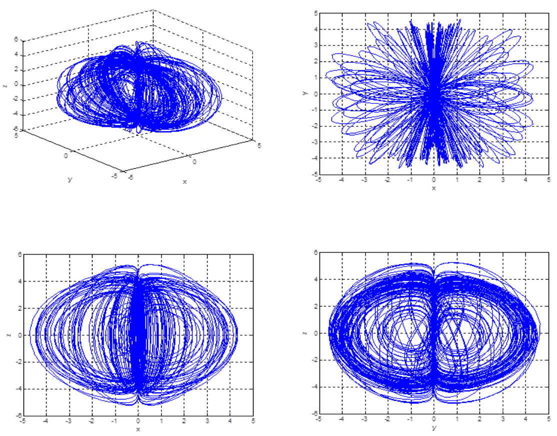

Fig.1. Phase trace diagram of the system in each plane

When the given initial value is (x (0), y (0), Z (0)) = (- 1, - 1, 4), the three Lyapunov exponents of the system are λ 1 "= 0.0386" λ 2 "= - 0.1232" λ 3 "= - 0.0968" by software simulation, and the formula can be used at the same time:

$$D_L = j + \frac{1}{\left|\lambda_{L_{j+1}}\right|} \sum_{i+1}^{j} \lambda_{L_i}$$

The dimension of the attractor is calculated as D_L = 2.18, which shows that the system appears chaos.

2.3. Characteristic analysis of system without balance point

(1) Symmetry

We define a new set of coordinates

$$\begin{cases} z_1 = -x \\ z_2 = -y \\ z_3 = z \end{cases} \qquad (a)$$

We found

$$\begin{cases} \dot{z}_1 = -ay - xz = az_2 + z_1 z_3 \\ \dot{z}_2 = bx - yz = -bz_1 + z_2 z_3 \\ \dot{z}_3 = 1 - x^2 - y^2 = 1 - z_1^2 - z_2^2 \end{cases} \qquad (b)$$

This means that the coordinate value (x, y, z) of the differential equation of the system is transformed into (- x, - y, z) without change, that is, the system has symmetry about the Z axis.

(2) Dissipation

$$\nabla \dot{V} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = 2z$$

When $\nabla \dot{V} < 0$, the system is always dissipative and converges with the index DV / dt = e ^ (- t). In this system, if $\nabla \dot{V} < 0$ is to be determined first, then with t $\rightarrow \infty$, each volume element including the system trajectory will shrink to 0. The asymptotic dynamic behavior of all system trajectories will be fixed in one attractor and limited to a limit subset of volume 0, which further proves the existence of attractors.

Based on the simulation of the system without balance point with different parameter values and the same initial value of [- 1-14], Fig.2. is obtained. These two figures are the sum of the three Lyapunov exponents of the system from - 5 to 5 after fixing one parameter. It can be seen that no matter which parameter is changed by the system, the sum cannot be stable at zero, so that It can be said that the system is not a conservative system.

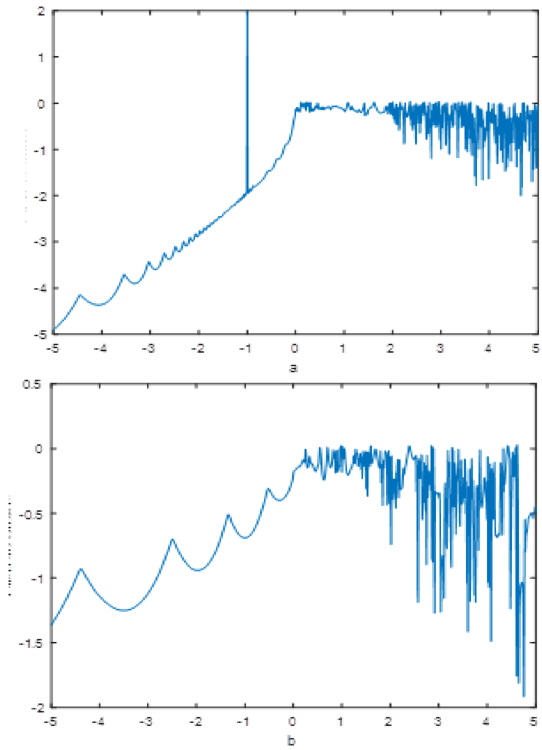

Fig.2. The sum of Lyapunov exponents with parameters a and b

(2) Equilibrium analysis

$0 = ay + xy$ (a)

$0 = -bx + yz$ (b)

$0 = 1 - x^2 - y^2$ (c)

for (a) and (b):

$xyz = -ay^2 = bx^2$ (d)

If $bx^2 + ay^2 = 0$ (e)

When a>0,b>0, $x = 0, y = 0$

Contradictory, that is, the system has no equilibrium point.

2.4. *Initial value sensitivity*

There are two parameters in the system, a = 0.05 is determined first, and the characteristic of parameter B is analyzed. The bifurcation diagram and Lyapunov exponent spectrum of the system are drawn with MATLAB software.

The initial values are [-1 -1 4] and [-2 -2 4], a = 0.05. Through software simulation, the following figures are obtained. From these two figures, it can be seen that the three-dimensional chaotic system without equilibrium point is in periodic state at b ∈ (- 2,0) and in chaotic state at b > 0, so it can be judged that the change of initial values has a very significant impact on the chaotic state of the system. The consequence is shown as Fig.3.

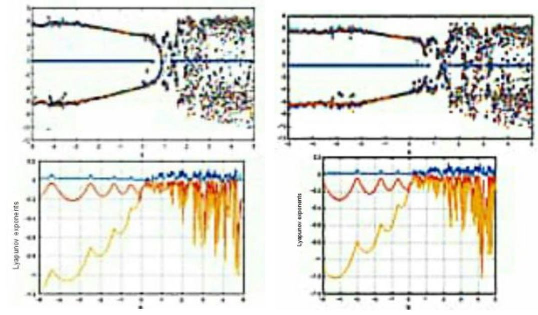

Fig.3. Bifurcation diagrams and Lyapunov exponents with initial values of [-1 -1 4] and [-2 -2 4]

3. Synchronous control analysis of a system without balance point

The following formula is defined as the driving system. By using observer based method, the following formula is transformed into another form, where $\mathbf{Ax}$ is the linear part, $\mathbf{B}f(x)$ is the nonlinear part, $\mathbf{G}$ is the constant part, and $g(x)$ is the output of the drive system [9].

The drive system is:

$$\begin{cases} \dot{x}_1 = ax_2 + x_1x_3 \\ \dot{x}_2 = -bx_1 + x_2x_3 \\ \dot{x}_3 = 1 - x_1^2 - x_2^2 \end{cases}$$

Transform:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}f(x) + \mathbf{G}$$

$$g(x) = \mathbf{Wx}$$

Namely:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \mathbf{B}\begin{bmatrix} x_1x_3 \\ x_2x_3 \\ x_1^2 + x_2^2 \end{bmatrix} + \mathbf{G}$$

It can be calculated:

$$\mathbf{A} = \begin{bmatrix} 0 & 0.05 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{G}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

The state observer is constructed as:

$$\dot{\mathbf{y}}=\mathbf{Ay}+\mathbf{B}f\left(x\right)+\mathbf{G}-\left(g\left(y\right)-g\left(x\right)\right)$$

Where $g\left(y\right)$ is the output of the state observer, and the error value $\mathbf{e}=\mathbf{y}-\mathbf{x}$ is set:

$$\dot{\mathbf{e}}=\dot{\mathbf{y}}-\dot{\mathbf{x}}=\mathbf{Ae}-\left(\mathbf{Wy}-\mathbf{Wx}\right)=\left(\mathbf{A}-\mathbf{W}\right)\mathbf{e}$$

When $(\mathbf{A}-\mathbf{W})$ is a time invariant matrix, select the appropriate gain matrix $\mathbf{W}$ so that the eigenvalue of $(\mathbf{A}-\mathbf{W})$ is less than zero, then the system gradually approaches stability.

The state observer system is:

$$\begin{bmatrix}\dot{x}_2\\\dot{y}_2\\\dot{z}_2\end{bmatrix}=\mathbf{A}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}+\mathbf{B}\begin{bmatrix}x_1x_3\\x_2x_3\\x_1^{\ 2}+x_2^{\ 2}\end{bmatrix}+\mathbf{G}-\mathbf{W}\begin{bmatrix}y_1-x_1\\y_2-x_2\\y_3-x_3\end{bmatrix}$$

The feedback gain matrix can be obtained by pole placement:

$$\mathbf{W}=\begin{bmatrix}1&0.05&0\\-1&1&0\\0&0&1\end{bmatrix}$$

The error system is:

$$\begin{bmatrix}\dot{e}_1\\\dot{e}_2\\\dot{e}_3\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix}$$

Because the eigenvalue of matrix $(\mathbf{A}-\mathbf{W})$ is less than zero, so the two systems are synchronized.

Next, we use MATLAB software to verify whether the state observer based synchronization method used in this paper can synchronize the system. Firstly, the Simulink module of MATLAB software is used to build the drive system and state observer system, and then the simulation is carried out.

Through simulation, the synchronization curve as shown in Fig. 4 is obtained. It can be seen from the diagram that the final output of the drive system and the state observer system are the same after a period of synchronization control under the condition of different initial values, which indicates that the applied state observer synchronization control method plays the role of synchronous control.

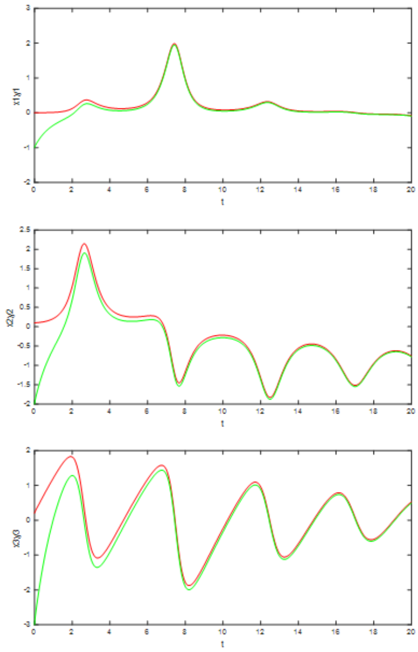

Fig.4. Synchronous curve

4. Conclusion

The full text is summarized as follows:

1. Through Lyapunov exponent spectrum, bifurcation diagram, phase trajectory diagram and sequence diagram, the hiding and coexistence characteristics of the system without equilibrium point are studied.
2. When the parameters are determined, a Lyapunov function is designed by using synchronization control method based on state observer. Finally, the simulation results are obtained through simulation, and the conclusion that the system can be synchronized by this method is drawn after analysis.

In the work of this paper, the basic dynamic characteristics and synchronous control of a system without balance point are studied preliminarily, but the research on the hidden characteristics and coexistence characteristics of the system without balance point is not deep enough, whether there are other hidden characteristics needs further study, and in the synchronous control, there are not enough methods, whether there are better methods Synchronization, which needs to be done in the next phase.

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Authors Introduction

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