

## Research Article

# Kinematics analysis and simulation of Manipulator by using Matlab/Simulink

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## ABSTRACT

The 6R robot can imitate the human arm to complete some target grabbing tasks, so the kinematics analysis of the robot is significant in scientific research and practical application. In this paper, a kinematics solution method of 6R robot based on analytic method is introduced, which is faster and more accurate in solution than the numerical method. Then the trajectory of the end effector is planned by using the quintic polynomial method, in this way, there are no sudden changes in the speed of the end effector of the robot, and the operation is more stable. Furthermore, the accuracy of the kinematics solution method is verified and the motion trajectory of the manipulator is simulated by Matlab. At last, the visualization of the robot kinematics model was realized based on the Simulink, and the kinematics simulation control system was established.

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## 1. Introduction

Today's society, to improve working conditions and production efficiency, the industrial robot has become an important tool. So that, the 6R robot is widely used in machining, electronic welding, industrial handling and other industries because of its flexibility and maneuver ability.

The kinematics research of 6R robot is the foundation. In this paper, by using D-H algorithm, the mathematical model of 6R robot is established and the kinematics problem is solved. The trajectory of the robot is simulated in joint space, and the smooth curves of angular displacement and angular velocity are obtained.

Furthermore, the rationality of the D-H parameters and Inverse kinematics results of the robot were verified by MATLAB, and the visualization of the robot kinematics model was realized based on the Simulink, and the kinematics simulation control system was established.

## 2. The Kinematics Analysis of the 6R Robot

### 2.1. D-H coordinate system

In this paper, the ABB-irb120 robot that shown in Figure 1 is used as an example of 6R robot in our discussion.

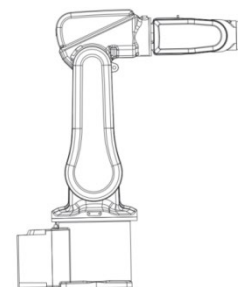


Fig. 1. The model of the 6R robot

The Denavit-Hartenberg matrix representation is used to describe the translational and rotational relationship

between the adjacent links. We established the coordinate system of each connected link, as shown in Figure 2, and we also obtained the D-H parameters, as shown in the Table 1.

In Figure 2, the coordinate system {0} is fixed on the base, as a defined reference coordinate system, and the coordinate system {1} usually coincides with the origin.

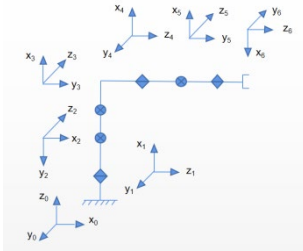


Fig. 2. Coordinate system of each link

Table 1. D-H Parameter Table

Joint i	$\theta_i$	$\alpha_i$	$a_i$	$d_i$
1	$\theta_1$	$-90^\circ$	0	290
2	$\theta_2$	0	270	0
3	$\theta_3$	$-90^\circ$	70	0
4	$\theta_4$	$90^\circ$	0	168
5	$\theta_5$	$-90^\circ$	0	0
6	$\theta_6$	0	0	0

## 2.2. Forward Kinematics Solution of the Robot

The purpose of the forward kinematics is to find out the position and posture of the end effector that relative to the coordinate system {0}. The position and posture between adjacent coordinate systems are represented by  $4 \times 4$  homogeneous transformation matrix as follows [1]:

$${}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

${}^{i-1}T_i$  represent the transformation matrix from 'the i-1 coordinate system to the i coordinate system. Through substituting the D-H parameters in Table 1 into formula (1), the homogeneous transformation matrix between each connecting rod can be obtained as follows:

$${}^0T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2 \cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2 \sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3 \cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3 \sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4T_5 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^5T_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The spatial posture and displacement transformation of the 6th coordinate frame that relative to the frame {0} can be get:

$${}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6 \quad (2)$$

## 2.3. Inverse Kinematics Solution of the Robot

If the parameters of each joint and link of the robot and the relative position of the end effector relative to the fixed reference coordinate system are known, how to get the angle  $\theta_i$  between the connected link of the robot, which is the inverse kinematics analysis solution.

Solution of  $\theta_1$ . According to the analysis of forward kinematics, we can multiply  ${}^0T_1^{-1}$  form the left on both side of the Eq. (2):

$${}^0T_1^{-1} {}^0T_6 = {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6 \quad (3)$$

By comparing the elements (3,4) of the Eq.(3),we can know that:

$$-p_y c_1 + p_x s_1 = 0 \quad (4)$$

where  $s_i \equiv \sin\theta_i$ ,  $c_i \equiv \cos\theta_i$ ,  $s_{ij} \equiv \sin(\theta_i + \theta_j)$ ,  $c_{ij} \equiv \cos(\theta_i + \theta_j)$ , so that:

$$\theta_1 = \text{atan2}(p_y - d_6 a_y, p_x - d_6 a_x) \quad (5)$$

Solution of  $\theta_2$ . Multiply  ${}^5T_6^{-1}$  form on both side of the Eq. (2) at the left:

$${}^5T_6^{-1} {}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 \quad (6)$$

By comparing the elements (1,4), (2,4), (3,4) of the Eq. (6),we can know that:

$$p_{5x} = p_x - d_6 a_x = c_1 (a_2 c_2 + a_3 c_{23} - s_{23} d_4)$$

$$p_{5y} = p_y - d_6 a_y = s_1 (a_2 c_2 + a_3 c_{23} - s_{23} d_4)$$

$$p_{5z} = p_z - d_6 a_z = d_1 - a_2 s_2 - a_3 s_{23} - d_4 c_2$$

In there, we let  $K = \sqrt{p_{5x}^2 + p_{5y}^2}$ ,  $L = p_z - d_6 a_z - d_1$ ,  $M = (L^2 + K^2 + a_2^2 - a_3^2 - d_4^2) / 2a_2$ , according to the trigonometric theorem,  $\theta_2$  can be obtained that [2]:

$$\theta_2 = \text{atan2}(K, L) - \text{atan2}(-M, (K^2 + L^2 - M^2)^{0.5}) \quad (7)$$

Solution of  $\theta_3$ . We can know that:

$$K - a_2 s_2 = a_3 s_{23} + d_4 c_{23};$$

$$L - a_2 c_2 = a_3 c_{23} + d_4 s_{23};$$

according to the trigonometric theorem,  $\theta_{23}$  can be obtained that:

$$\begin{aligned}\theta_{23} &= \text{atan2}(a_3, d_4) - \text{atan2}(E, (a_3^2 + d_4^2 - E^2)^{0.5}) \\ E &= K - a_2 * \cos \theta_2 \\ \theta_3 &= \theta_{23} - \theta_2\end{aligned}\quad (8)$$

Solution of  $\theta_4$ . Multiply  ${}^0T_3^{-1}$  from the left on both side of the Eq. (2) :

$${}^0T_3^{-1} {}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3$$

$$W = -s_4 s_5 = a_x s_1 - a_y c_1$$

$$Z = -c_4 s_5 = c_1 c_{23} a_x - s_{23} a_z + s_1 c_{23} a_y$$

So that, we can get the angle  $\theta_4$ :

$$\theta_4 = \text{atan2}(W, Z); \quad (10)$$

Solution of  $\theta_5$ . Multiply  ${}^0T_4^{-1}$  from the left on both side of the Eq. (2)

$${}^0T_4^{-1} {}^0T_6 = {}^4T_5 {}^5T_6 \quad (11)$$

By comparing the elements of the Eq. (11), we can know that:

$$s_5 = -a_x (s_1 s_4 + c_1 c_4 c_{23}) - a_y (c_1 s_4 - s_1 c_4 c_{23}) - a_z c_4 s_{23}$$

$$c_5 = -a_x c_1 s_{23} - a_y s_1 s_{23} - a_z c_{23}$$

$$\theta_5 = \text{atan2}(s_5, c_5) \quad (12)$$

Solution of  $\theta_6$ . It is similar to the solution of  $\theta_5$ , we just need to multiply  ${}^0T_5^{-1}$  from the left on both side of the Eq. (2)

$${}^0T_5^{-1} {}^0T_6 = {}^5T_6 \quad (13)$$

By comparing the elements of the Eq. (13), we can know that:

$$s_6 = -n_x (s_1 c_4 + c_1 s_4 c_{23}) - n_y (c_1 c_4 + s_1 s_4 c_{23}) + n_z s_4 s_{23}$$

$$c_6 = o_x (c_4 s_1 - c_1 s_4 c_{23}) - o_y (c_1 c_4 + s_1 s_4 c_{23}) + o_z s_4 s_{23}$$

$$\theta_6 = \text{atan2}(s_6, c_6) \quad (14)$$

## 2.4. Trajectory planning of the 6R robot

In this paper, we use the method of quintic polynomial interpolation to plan the trajectory of the 6R robot.

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

So that, the function expression of the velocity is:

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

And the function expression of the acceleration is:

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

By making constraints about the position, angular velocity and angular acceleration of each joint in space at the starting and ending positions of the path point:

$$\begin{cases} \theta(t_0) = a_0 \\ \theta(t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 + a_4 t_f^4 + a_5 t_f^5 \\ \dot{\theta}(t_0) = a_1 \\ \dot{\theta}(t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2 + 4a_4 t_f^3 + 5a_5 t_f^4 \\ \ddot{\theta}(t_0) = 2a_2 \\ \ddot{\theta}(t_f) = 2a_2 + 6a_3 t_f + 12a_4 t_f^2 + 20a_5 t_f^3 \end{cases}$$

So this is obtained from the above formula:

$$\begin{cases} a_0 = \theta_0 \\ a_1 = \dot{\theta}_0 \\ a_2 = \frac{\ddot{\theta}_0}{2} \\ a_3 = \frac{20\theta_f - 20\theta_0 - (8\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3} \\ a_4 = \frac{30\theta_f - 30\theta_0 - (14\dot{\theta}_f + 16\dot{\theta}_0)t_f + (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2}{2t_f^3} \\ a_5 = \frac{12\theta_f - 12\theta_0 - (6\dot{\theta}_f + 6\dot{\theta}_0)t_f - (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3} \end{cases}$$

The quintic polynomial function method can be obtained, by substituting the above coefficients into the basic expression,

## 3. The Kinematics Simulation Verification

We use the Matlab Robotics Toolbox to verify the accuracy of kinematics solution. By given a joint angle group 'q=[pi/3, pi/4, 3\*pi/4, -pi/5, pi/5, pi/6]', we use the forward and inverse kinematics algorithm finished in chapter 2.2 and 2.3 to get the pose and posture matrix of end effector, and the solution of each joint angle in Matlab, as shown in Figure 3.

Then by comparing with the result that obtained by using the 'Fkine' and 'Ikine' function of Matlab, we found that the two results are consistent, so that the kinematics solution is verified.

T =					
0.6235	0.2091	-0.7534	0.06081		
-0.5528	-0.5636	-0.6139	0.1053		
-0.5529	0.7992	-0.2358	-0.06921		
0	0	0	1		
-1.0471	1.7033	-2.3521	-2.7622	-1.9400	-1.9437
-1.0471	0.7854	2.3602	-0.3637	-1.8126	0.9640
-1.0471	1.7033	-2.3521	1.5907	-1.2855	-2.1235
-1.0471	0.7854	2.3602	1.7318	-1.8082	2.0203
-1.0471	1.7033	-2.3521	0.3794	1.9400	1.1979
-1.0471	0.7854	2.3602	2.7779	1.8126	4.1056
-1.0471	1.7033	-2.3521	3.1217	1.2855	1.0181
-1.0471	0.7854	2.3602	4.8734	1.8082	5.1619

Fig. 3. Kinematics solution result finished by Matlab

The trajectory planning of the robot is also simulated by Matlab, the curve of the velocity and acceleration of each joint angle with time, shown in the Figure 4.

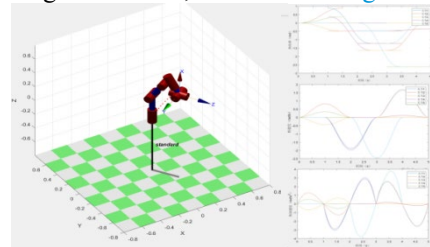


Fig. 4. Trajectory path and joint motion characteristic curve

#### 4. The Visual simulation

By establishing the model based on SolidWorks 2016. First, the models of the robot is built and assembled. Then the assembly model is converted into the format of. urdf and imported into the Simulink. By referring with the previous solving function, and by using the modular simulation interface of the Simulink, the visual simulation framework of the robot is finished, shown in Figure 5 and Figure 6.

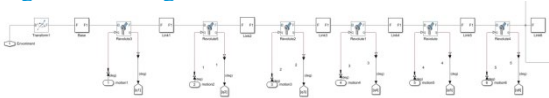


Fig. 5. The model of robot imported in Simulink

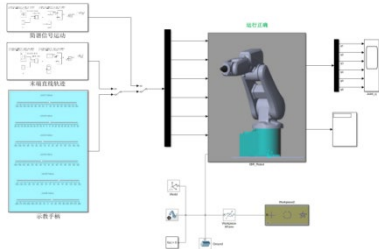


Fig. 6. Kinematics visual simulation by Simulink

#### 5. Conclusion

In this paper, the IRB120 robot was used as the research object, using the D-H method to establish the connecting rod coordinate system, the solution of the kinematics analysis was finished. Then the model of the robot is established by the Matlab. From the simulation results, we can see that the model of the robot established in this study is correct. And the trajectory planning of the end effector is carried out, and the change of the angle of each joint with time is recorded, which verified the effectiveness of the kinematics algorithm.

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#### Authors Introduction

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He is an associate professor, Beijing Jiaotong University. His research interest is Intelligent Robot, Machine Vision, and Image Processing.

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