# Research Article <br> Inter－Induce computation and its Philosophical Foundation 

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#### Abstract

Set theory is based on the distinguishability of elements．How to recognize and identify the world is the essence of set theory．If each element cannot be identified，all the elements are one set．So the set does not make sense．The Heart Sutra is highly rational and can be interpreted mathematically．The mathematical interpretation of the Heart Sutra shows the divergence of how to discriminate．Based on this world view of Heart Sutra，we propose Inter－Induce computation， IIC as a novel calculation paradigm that does not depend on set theory．This paper gives an overview and philosophical foundation of IIC．


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## 1．Introduction

There are several types of the calligraphy of a circle（Enso 円相 in Japanese），for example，Zen master Ikkyu painted a perfect circle（Fig． 1 left），while Zen master Hakuin or Buddhist preast Sengai drowned not a perfect circle，but the circle is broken in one place（Fig． 1 right）．

We do not know the beginning and end of a perfect circle，but if the circle is broken in one place， we know the point of beginning or the end．And we know the beginning and end of a broken circle from brushstroke（Fig． 1 right）．

Topologically，a perfect circle with one point broken is a straight line．It creates＂time＂because a straight line has a beginning and an end．In other words，when a perfect circle is broken in one place， time is created．The perfect circle is a time－ integrated dynamical system，and breaking the perfect circle causes the dynamical system to evolve in time．The phenomenological equations of the dynamical system are known，but the phenomenological equations of this world are unknown．


Fig．1．Example of calligraphy of circles，left）Zen Master Ikyuu（Edo period）created a complete circle calligraphy）right） many calligraphy of circles are not complete circle，whichhas a ＂start point＂as breaking point．

## 1．2．Heart sutra

The Heart Sūtra is a popular sutra in Mahāyāna Buddhism．In Sanskrit，the title Prajñāpāramitāhrdaya translates as＂The Heart of the Perfection of Wisdom＂ ［1］）．Below is the first part of the sutra；
［．．．］
色不異空 空不異色，
色即是空 空即是色，

## ［．．．］

where 色 means substances［2］），不異，＂not different＂ ［3］），空，＂emptiness＂and 即是，＂exactly＂［4］）．Hence，an English translation of this part is as follows；
［．．．］
Substances are not different from Emptiness
Emptiness is not different from Substances
Substances are exactly Emptiness
Emptiness is exactly Substances
［．．．］

For
Substances are not different from Emptiness and Emptiness is not different from Substances，

By denoting＂not different from＂as the binary relation R， we obtain binary relations；
［．．．］
s Re and e R s，
［．．．］
where s stands for＂substances＂，e，＂emptiness＂；we call the relation R as H relation， $\mathrm{R}_{\mathrm{H}}$ ．

Proposition $1 \mathrm{R}_{\mathrm{H}}$ relation is equivalence relation．
The equivalence relation is mathematically satisfied by；
i）$x R x$（Reflexivity），
ii）$x R y$ and $y R x$（Symmetry），
iii）$x R y$ and $y R z$ implies $x R z$（Transitivity）．
i）Reflexivity：since substances＂are not different from＂ substances and＂emptiness is not different from emptiness，＂it is obvious（note that $\mathrm{R}_{\mathrm{H}}$ stands for the binary relation of＂x not different from $\mathrm{y}=\mathrm{x}_{\mathrm{H}} \mathrm{y}$＂）．
ii）Symmetry：obvious．Because， $\mathrm{R}_{\mathrm{H}}$ relation is the statement of the symmetric relation on substances and emptiness．
iii）Transitivity：from i）s $R_{H} s$ and ii）claims that $s R_{H} e$ and $e$ and $R_{H} s$ ，if $R_{H}$ does not satisfy transitivity s $R_{H} e$ and e $R_{H} s$ does not fulfill $s R_{H} s$ ，which contradicts to i） and ii）．

Heart sutra is based on $\mathrm{H}_{\mathrm{R}}$ and argues that there is no substance in perception with time development．The senses perceive time change；Heart sutra asserts that there is no substance in the perception of such time change．To recognize time change，Heart sutra affirms that time exists．The calligraphy of the circle suggests that symmetry breaking creates time．When time arises，time
change is created．We perceive time change；Heart sutra suggests that the perception created by time change has no substance．

## 2．Method

We consider computations based on the philosophy of Heart sutra；we give basic concepts and notations．
A computing system C is composed of collection of states，S，computing maps，M．We denote the set of states as $S$ and $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{s}_{\mathrm{i}}$ or $\mathrm{s}_{\mathrm{j}}$ are element in S ；the lowercase of s denotes only one state in S and different suffix denotes different state． M stands for the set of computing maps and for denoting computing maps we will use the lowercase of $\mathrm{f}, \mathrm{g}, \mathrm{h}$ ，we call such computing map as ＂computing map．＂

A computing map has domain and codomain and transforms from domain to codomain．An arrow denotes a computing map as $f: a \rightarrow b$ ，where $a$ is the domain and $b$ is the codomain；the operation $\operatorname{dom}(\mathrm{f})$ gives $\mathrm{a}, \operatorname{cod}(\mathrm{f})$ gives $b$ ．For computing maps $f, g$ ，in case $\operatorname{cod}(f)=\operatorname{dom}(g)$ ， which is denoted as $g \circ f$ and called composition $f$ and $g$ ； －denotes composition of computing maps．For all computing maps $\mathrm{f}: \mathrm{a} \rightarrow \mathrm{b}, \mathrm{g}: \mathrm{b} \rightarrow \mathrm{c}$ ，if a computing map Ib gives， $\mathrm{Ib} \circ \mathrm{f}=\mathrm{f}$ and $g \circ \mathrm{Ib}=g$ ， Ib is called identity computing map；we assume that every computing map has identity computing map．Below the word，computing map（s）include（s）identity computing map（s）．A computing is a composition of computing maps，where the number of compositions is greater of equal to zero． Things that computing maps can operate are the states of the computing system，while things computing maps cannot operate are not the states of the system．

Definition of Algorithm：A sequence of computing maps from $\mathrm{s}_{\mathrm{i}}$ to $\mathrm{s}_{\mathrm{j}}$ is called algorithm，where $\mathrm{s}_{\mathrm{i}}$ and $\mathrm{s}_{\mathrm{j}}$ ；note that an algorithm of $f \circ g$ and $g \circ f$ is different，because they are different sequence．We denote sequence of computing maps from $\mathrm{s}_{\mathrm{i}}$ to $\mathrm{s}_{\mathrm{j}}$ as $\mathrm{s}_{\mathrm{i}} \rightarrow \mathrm{s}_{\mathrm{j}}$ ．
Definition of Programming：Programming is defined as changing the order of the sequences of computing maps．

## 3．Result

We consider computation without concept of absolute time，quantity and quality that Heart sutra claims．
Let us consider the case where there are more than two computational systems．We assume that the computational systems are autonomous and do not interact with each other，so we will consider a system
where more than two computational systems interact with each other.
In the following we will consider the interaction between two computing systems between $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$. We conjecture those interactions of more than two computing systems requires careful consideration.

Definition of Interaction: Let $f_{i}$ denote maps of $C_{i}, g_{j}$, of $\mathrm{C}_{\mathrm{j}}$. If the codomain of $\mathrm{f}_{\mathrm{i}}$ is contained in the domain of $g_{j}$, we define the composition of maps, $g_{j} \circ f_{i}$ is the interaction from $C_{i}$ to $C_{j}$. So, $f_{i} \circ g_{j}$ are the interaction from $\mathrm{C}_{2}$ to $\mathrm{C}_{1}$.
$f_{i}$ or $g_{i}$ is a notation for a collection of maps, not only a single map. In $f_{i} \circ g_{k}, f_{i}$ and $g_{k}$ denote collection of maps such that they can compose maps between $f_{i}$ and $g_{k}$. Let $\mathrm{C}_{1}$ has map $\mathrm{m}_{\mathrm{i}}: \mathrm{a} \rightarrow \mathrm{b}, \mathrm{m}_{\mathrm{j}}: \mathrm{a} \rightarrow \mathrm{c}$ and $\mathrm{m}_{\mathrm{e}}: \mathrm{j} \rightarrow \mathrm{b}$; and $\mathrm{C}_{2}$ has $\mathrm{O}_{\mathrm{i}}: \mathrm{b} \rightarrow \mathrm{d}, \mathrm{O}_{\mathrm{k}} \mathrm{b} \rightarrow \mathrm{h}$ and $\mathrm{O}_{\mathrm{p}}: \mathrm{a} \rightarrow \mathrm{g}$. Then $\mathrm{f}_{1}$ is composed of $m_{i}$ and $m_{e}$, because the codomain of $m_{i}$ and $\mathrm{m}_{\mathrm{e}}$ are included by the domain of $\mathrm{O}_{1}$ and $\mathrm{O}_{\mathrm{k}}$. Hence, $\mathrm{f}_{1}=$ $\left\{\mathrm{m}_{\mathrm{i}}, \mathrm{m}_{\mathrm{e}}\right\}, \mathrm{g}_{1}=\left\{\mathrm{O}_{\mathrm{i}}, \mathrm{O}_{\mathrm{k}}\right\}$, where $\{$,$\} denotes a collection of$ maps.

Definition of Strength of Interaction: We define the magnitude of the overlap between the collection of codomain of $f_{1}$ and the domain of $g_{1}$ as the "strength of the interaction". If the overlap is the empty, we define the interaction as 0 , or none.

Definition of Inter-Induce Computation, IIC: The composite sequence of maps of $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$ is defined as Inter Induce Computing, IIC or IIC $<\mathrm{C}_{\mathrm{i}}, \mathrm{C}_{\mathrm{j}}>$; the case where the composite sequence consists of only the interaction of $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$ is called pure IIC. In this paper, we consider only pure IIC, and for simplicity, we refer to regular IIC as IIC in the following.
We show examples of pure IIC and irregular IIC, below we will omit o to denote composition of maps; so $f_{i} \circ g_{j} \circ$ $\mathrm{f}_{\mathrm{m}} \circ \mathrm{g}_{\mathrm{n}} \circ \mathrm{f}_{\mathrm{k}} \circ \mathrm{g}_{\mathrm{k}} \equiv \mathrm{f}_{\mathrm{i}} \mathrm{g}_{\mathrm{j}} \mathrm{f}_{\mathrm{m}} \mathrm{g}_{\mathrm{n}} \mathrm{f}_{\mathrm{k}} \mathrm{g}_{\mathrm{k}}$;
regular IIC: $f_{i} g_{j} f_{m} g_{n} f_{k} g_{k} f_{m} \ldots$,
irregular IIC: $\underline{f}_{\underline{i}} f_{f_{j}} f_{\underline{m}} g_{n} f_{k} g_{k} g_{\underline{n}} \ldots$, underlined compositions illustrate irregular IIC.

Corollary 1 An irregular IIC can be transformed to a regular IIC.

For the irregular sequence $f_{i} f_{j} f_{m} g_{n} f_{k} g_{k} g_{n}$, by setting $F_{i}$ $f_{i} f_{j} f_{m}$ and $G_{k}=g_{k} k_{n}$, the sequence is transformed to regular IIC, $\mathrm{F}_{\mathrm{i}} \mathrm{g}_{\mathrm{m}} \mathrm{f}_{\mathrm{m}} \mathrm{G}_{\mathrm{k}}$.

Definition of Halt of IIC: When a IIC has zero interaction, the IIC halts. We name the number of
compositions of a IIC as length of IIC or composition sequences.

For a collection of maps $M_{k}$, we denote the domain and codomain of $\mathrm{M}_{\mathrm{K}}$ are respectively, $\operatorname{dom}\left(\mathrm{M}_{\mathrm{k}}\right)$ and codom $\left(\mathrm{M}_{\mathrm{k}}\right)$.

Corollary 2 In regular IIC of $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$, if $\operatorname{codom}\left(\mathrm{M}_{\mathrm{i}}\right)=$ $\operatorname{dom}\left(\mathrm{M}_{\mathrm{j}}\right)$ and $\operatorname{codom}\left(\mathrm{M}_{\mathrm{j}}\right)=\operatorname{dom}\left(\mathrm{M}_{\mathrm{i}}\right)$ then IIC does not halt, where $M_{i}$ and $M_{j}$ are respectively collection of maps of $C_{i}$ and $C_{j}$.

If the IIC halts, there exist a map $h$ in $\mathrm{C}_{\mathrm{i}} / \mathrm{C}_{\mathrm{j}}$ such that codom(h) does not in the $\operatorname{dom}(\mathrm{J})$ in $\mathrm{C}_{\mathrm{j}} / \mathrm{Cj}$. This contradicts the condition of corollary 2.

Proposition 4 In any IIC of $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$, if $\operatorname{codom}\left(\mathrm{M}_{\mathrm{i}}\right)=$ $\operatorname{dom}\left(\mathrm{M}_{\mathrm{j}}\right)$ and $\operatorname{codom}\left(\mathrm{M}_{\mathrm{j}}\right)=\operatorname{dom}\left(\mathrm{M}_{\mathrm{i}}\right)$ then IIC does not halt, where $M_{i}$ and $M_{j}$ are respectively collection of maps of $C_{i}$ and $C_{j}$.

By corollary 1, any irregular IIC can be transformed to regular IIC. And by corollary 2, if the condition of proposition 4 is satisfied, IIC does not halt.

## 4. Conclusion

In this computational framework, each agent cannot know internal state of others. But, can "implement" algorithm in the interactions. Each agent induces action of others.
We obtained a condition, such interactive computations does not halt, which means, eternal.

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