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Research Article

Creation and Testing of a Conservative Chaotic Multi-Scroll System

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ABSTRACT

The multi-scroll conservative chaotic system represents a novel category of chaotic systems renowned for their complex dynamics. Due to their intricate behavior, they have garnered significant interest among researchers in the field. This study focuses on the creation of a conservative multilayer chaotic system. The design involves integrating a polygon-free sinusoidal function to generate multiple scrolls within this conservative multilayer chaotic system. By incorporating nonlinear functions, this system generates one-dimensional linear multi-roll and two-dimensional multi-roll lattice distributions.

1. Introduction

Since the 20th century, nonlinear science has developed rapidly, and it mainly includes three major aspects: fractal, chaos and soliton. Among these three aspects, chaos has attracted the attention of many scholars because of its particularity. The multi-scroll conservative chaotic system stands out for its intricately woven dynamical properties, garnering heightened scholarly attention as a distinctive breed of chaotic systems.

Researchers are increasingly delving into the creation of multi-scroll chaotic systems characterized by unique and captivating morphologies. This is driven by the sheer complexity of their dynamics, coupled with their propensity for wider spectral width when compared to their counterparts. Although somewhat less explored due to limitations in analytical methods, these multi-scroll chaotic systems find real-world applications in realms such as communication encryption and image encryption. Their distinctive qualities, exhibited by multi-scroll chaotic attractors [1], assume a central role in these practical applications. Investigating these systems not only enriches our comprehension of the fundamental mechanisms underpinning chaotic attractors but also fosters innovation in the design of new chaotic systems.

2. Creation and Testing of a Conservative Chaotic Multi-scroll Syste

This paper delves into the exploration of the multi-scroll conservative chaotic system. The creation of multiple scrolls within the conservative chaotic system is accomplished through the application of the non-double angle sine function. The analysis and examination of the Lyapunov exponential map, the bifurcation diagram, and the attractor phase diagram serve as essential tools for unraveling the dynamic properties of the multi-scroll attractor.

The Lyapunov exponential map reflects several Lyapunov indices of a chaotic system through images, and obtains the motion state of the system with the change of parameters, or the motion state of the system with the change of initial value through the comparison of zero. If there is a line on a certain section of the bifurcation diagram, it indicates that the motion generated by the system is periodic; If there are two lines, then the motion of the system is period-double; If the image is a pair of chaotic points, then the movement of the system is periodic. The phase diagram can intuitively see the motion of the system. The phase diagram mainly reflects whether the

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system can produce chaotic motion and whether the motion trajectory in the phase space is chaotic.

2.1. *The advent of multi-scroll chaotic systems*

The four-dimensional structure investigated in this study represents a conservative chaotic model, whose dynamic equation x is assigned to sinx. At the same time, given $\dot{x} = 0, \dot{y} = 0, \dot{z} = 0, \dot{w} = 0$, and the resulting equation is shown in Eq, (1).

$$\begin{cases} 0 = ay + byw \\ 0 = -\text{asin}\,(x) \\ 0 = cw \\ 0 = -bsin(x)y - cz \end{cases} \quad (1)$$

The Jacobian matrix [2] of the equilibrium point is shown in Eq. (2).

$$J(x) = \begin{bmatrix} 0 & a + bw & 0 & by \\ -a\cos(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & c \\ -bcos(x)y & -bsin(x) & -c & 0 \end{bmatrix} \quad (2)$$

Order $J - \lambda E = 0$,when $\lambda_1 = -ai, \lambda_2 = ai, \lambda_3 = -ci, \lambda_4 = +ci, a > 0, b > 0, c > 0$, Such equilibrium points are central points, When $\lambda_1 = a, \lambda_2 = -a, \lambda_3 = -ci, \lambda_4 = +ci, a > 0, b > 0, c > 0$, Such equilibrium points are called unstable saddle point [3].

The eigenvalues of the equilibrium point and corresponding types are shown in Table 1.

Table 1. Characteristic value and type of balance point

balance point (x_0, y_0, z_0, w_0)	characteristic value $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$	type
$(2k\pi, 0,0,0)$	$(-ai, ai, -ci, +ci)$	Center point
$((2k + 1)\pi, 0,0,0)$	$(a, -a, -ci, +ci)$	Unstable saddle point

Triangular wave, sawtooth wave, time delay function and sine function without double Angle are often used to generate multi-scroll. In this paper, the sine function without double Angle is used to generate multiple volumes.

2.2. *Foundational Investigation of the dynamics of a Multi-Scroll Chaotic System*

Taking the initial value as varying, is $[\pi/2 + 2k\pi, 1,1,1], k = (0,1,2)$, when k=0, the initial value is [π/2,1,1,1], let the parameters a=9, c=14, the Lyapunov exponent of the system equation is displayed in Fig. 1.

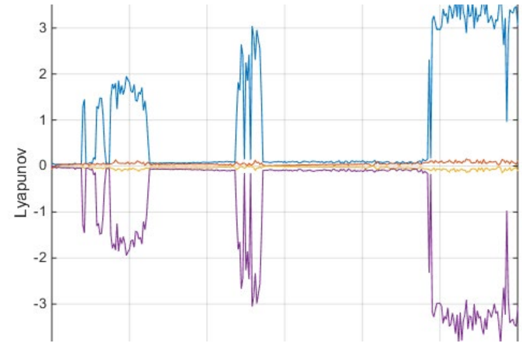

Fig.1. Lyapunov exponent diagram when a=9, c=14

Generated bifurcation diagram corresponding to Lyapunov exponential diagrams. When parameters a=9, c=14 is shown in Fig. 2.

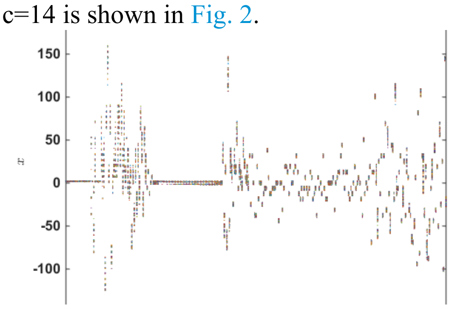

Fig.2. Bifurcation Diagram

When k=0, the initial value is [π/2,1,1,1], and the parameter a=4, c=4, the Lyapunov exponent diagram of the system equation is displayed in Fig. 3.

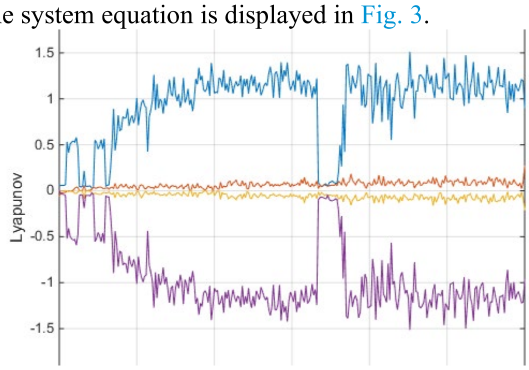

Fig.3. Lyapunov exponent diagram when a=4, c=4

Generated bifurcation diagram corresponding to Lyapunov exponential diagrams. when Parameters a=4, c=4 is shown in Fig. 4.

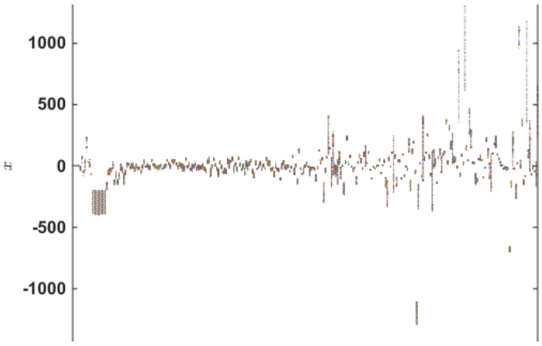

Fig.4. Bifurcation Diagram

When the parameters alter, the distribution of the Lyapunov exponent diagram will undergo significant changes. The system can produce chaos over a wide range of parameter values.

2.3. *A multi-scroll attractor and its stability analysis*

The span within which the system exhibits chaos is evidently affected by a parameter adjustment. In this scenario, altering the system's initial value unveils diverse phase diagrams, and by manipulating the phase diagram, one can scrutinize the attractor attributes of the system.

Take the system parameter $a = b = c = 4$, then only the initial value of the system is changed.

Let the initial value $y_0 = z_0 = w_0 = 1$, and the Lyapunov index diagram changing with the initial value x is displayed in Fig. 5.

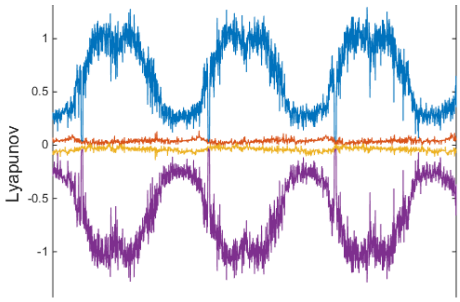

Fig.5. Lyapunov exponent diagram

The corresponding bifurcation diagram is shown in Fig. 6.

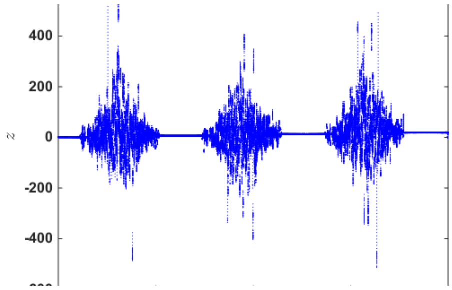

Fig.6. Bifurcation Diagram

It is clear that the range of chaos produced by the system is periodic by contrasting the bifurcation diagram in Fig. 6 with the Lyapunov exponent diagram in Fig. 5 of the figures. The shift in Lyapunov exponent inside this range of $[0, 2\pi]$ resembles the representation of a sine function.

When $k = 0$, the initial value is $[\pi/2,1,1,1]$, and the 16-scroll chaotic attractors is shown in Fig. 7.

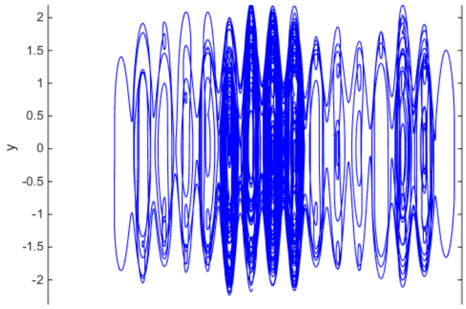

Fig.7. 16-scroll chaotic attractors

When $k = 1$, the initial value is $[5\pi/2,1,1,1]$, and the 16-scroll chaotic attractors is shown in Fig. 8.

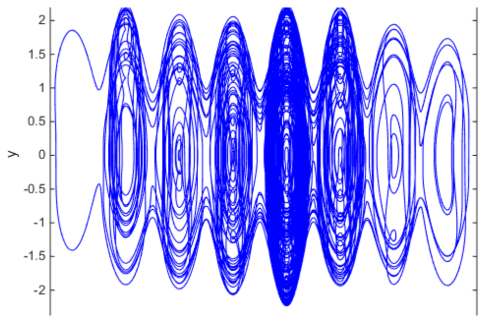

Fig. 8. 8-scroll chaotic attractors

When $k = 2$, the initial value is $[9\pi/2,1,1,1]$, and the 16-scroll chaotic attractors is shown in Fig. 9.

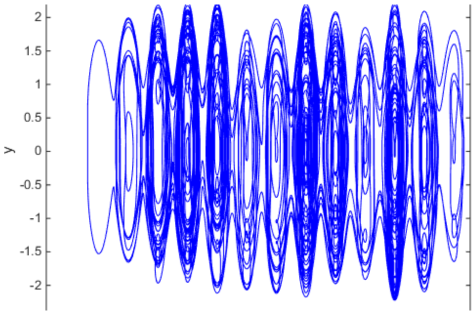

Fig.9. 13-scroll chaotic attractors

As can be concluded from Fig. 7, 8 and 9, the amount of scrolls in the x-y phase diagram varies dramatically with the modification of the initial value, indicating that the system is multi-stable and seems to be conservative. The multi-volume phase diagram indicates that the multi-stability is confirmed and the one-dimensional multi-scroll chaotic attractor is obtained by injecting sin (x). To enable the system to produce multi rolls, the non-double angle sine function is introduced for y, z, and w.

When $a = b = c = 4$, the eigenvalues of the equilibrium point and their corresponding types are shown in Table 2.

Table 2. Characteristic value and type of balance point

balance point (x_0, y_0, z_0, w_0)	characteristic value $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$	type
$(2k\pi, 2k\pi, 0,0)$	$(-4i, 4i, -4i, +4i)$	Center point
$(2k\pi, (2k+1)\pi, 0,0)$	$(4, -4, -4i, +4i)$	Unstable saddle point
$((2k+1)\pi, 2k\pi, 0,0)$	$(4, -4, -4i, +4i)$	Unstable saddle point
$((2k+1)\pi, (2k+1)\pi, 0,0)$	$(-4i, 4i, -4i, +4i)$	Center point

Where N is the gate that limits the sine function, and its action controls the width. When different functions are used in different cases, f (u) is linear when $u < -N$ and $u > N$, and f (u) is a nonlinear function only when $-N \leq u \leq N$, the system equation can produce multiple volumes. When $N = 2n\pi$, the equation of the nonlinear function [4] is shown in Eq. (3).

$$f(u) = \begin{cases} u + N, u < -N \\ \sin(u), -N \leq u \leq N \\ u - N, u > N \end{cases} \qquad (3)$$

According to this nonlinear function, the Lyapunov exponent diagram with different initial values can be obtained, as shown in Fig. 10.

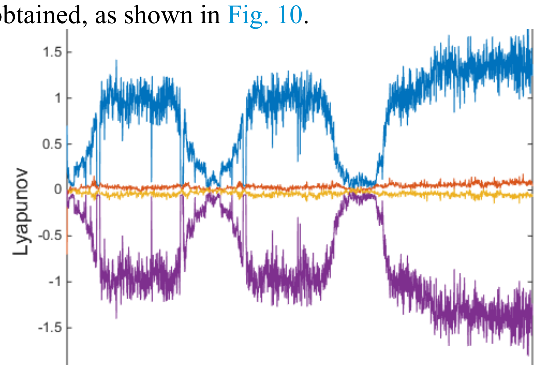

Fig.10. Lyapunov exponent diagram

In Fig. 10, it can be seen that the system before $x = 4\pi$. The Lyapunov exponent of the system that is larger than zero nonetheless follows a change rule that is roughly periodic.After $x = 4\pi$, the system because of the use of two multiplicative angle sine function, the law of change has changed, breaking the rule of change before. The graphic illustrates the Lyapunov exponent's trend of slow expansion following 4π.

Here, the initial value is $[\pi/2, \pi/2, 1, 1]$,and the parameter is $a = b = c = 4$, and the two-dimensional multi-volume attractor phase diagram can be obtained. and the amount of attractors is associated with the taken by the n. If the amount of scroll attractors produced by the system be M , then there is Eq. (4).

$$M = (2n)^2 + (2n+1)^2 \qquad (4)$$

Let the order of the two-dimensional multivolume it generates be R, the order Eq. (5) is available.

$$R = 4n + 1 \qquad (5)$$

Taking n=1, in other words, $N = 2\pi$,the phase diagram of the generated 5×5 multi-scroll chaotic attractor phase diagram is shown in Fig. 11.

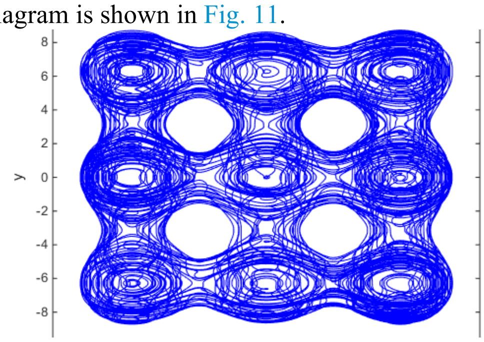

Fig.11. 5×5 multi-scroll chaotic attractor phase diagram

Fig. 11 shows that the number of attractors is $M = (2)^2 + (2+1)^2 = 13$.

Taking n=2, in other words, $N = 4\pi$, the phase diagram of the generated 9×9 multi-scroll chaotic attractor phase diagram is shown in Fig. 12.

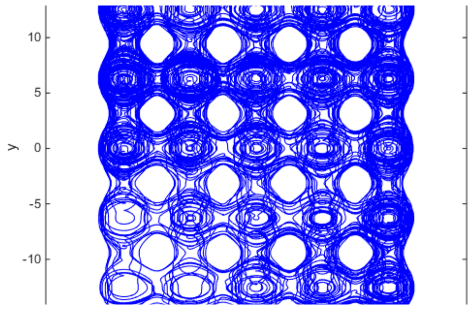

Fig.12. 9×9 multi-scroll chaotic attractor phase diagram 1

Keep n=2 unchanged, alter the initial value to $[\pi/2, 5\pi/2, 1, 1]$, the phase diagram of the generated 9×9 multi-scroll chaotic attractor phase diagram is shown in Fig. 13.

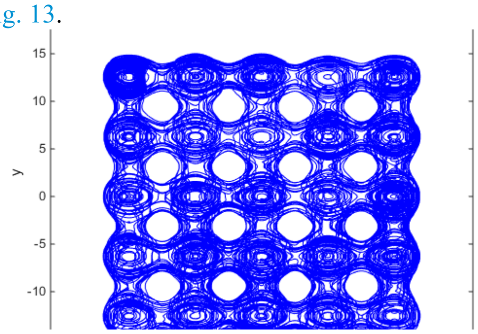

Fig.13 9×9 multi-scroll chaotic attractor phase diagram 2

By modifying the initial values, two distinct phase diagrams of multi-scroll attractors were obtained, as illustrated in Fig. 12 and Fig. 13. A comparative analysis of these two diagrams revealed that changing the initial values had a notable impact on the internal motion trajectory within the phase diagram. However, it's noteworthy that the number of scrolls in the attractor remained constant, unaffected by these changes.

3. Conclusion

This paper focuses on the conservative chaotic system and explores the application of the sine function without double angles as the nonlinear function in this context. By altering the selection and quantity of variables related to this nonlinear function, the research investigates the generation patterns of multi-scroll chaotic attractors. The validation and characterization of both the multi-scroll conservative chaotic system and the principles governing the multi-scroll chaotic attractor are conducted through MATLAB simulations.

The primary objective of this paper is the integration of the angle-free sine function within the conservative system, resulting in the derivation of a multi-scroll conservative chaotic system—this serves as the central research endeavor. Through the adjustment of nonlinear function introduction, the paper successfully achieves the generation of one-dimensional and two-dimensional chaotic attractors while also confirming the multistability of the system.

The research in this paper has verified some theories and achieved certain results, but there are still many shortcomings to be corrected and a lot of work to be completed in the research process, as follows:

The sinusoidal function without multiple angles used in this paper is too simple and can only be used as the basic switching function. Moreover, the chaotic attractor obtained is too simple, with only one and two-dimensional mesh-like structures. It is necessary to further deepen the use of the switching function, so that attractors with more complex structures can be generated, and three-dimensional attractor topologies can be made on the basis of two-dimensional. At the same time, we can study the application of sinusoidal function in hyperchaotic system or higher dimensional chaotic system, so as to construct more complex and characteristic chaotic attractors.

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Authors Introduction

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In 2021, he attained his Bachelor of Engineering degree from Tianjin University of Science and Technology, China, majoring in the School of Electronic Information and Automation. Currently, he is enrolled in a master's program in engineering at the same university, continuing his academic journey.

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