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Research Article

Efficient Movement of Formation Transformation of Multiple Mobile Robots on a Grid Plane

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ABSTRACT

Multi-robot motion management is an important issue in the research of mobile robotics. Formation transformation of multi-robots is one of the problems of multiple mobile-robots locomotion. The algorithms proposed in the past for this problem cannot guarantee the best solution. From a mathematical perspective, this paper uses singularity check for a matrix and Munkres assignment algorithm to develop a new algorithm for solving this problem. Comparing to the Monte Carlo method, the new algorithm can have better, at least the same, results in a very short time.

1. Introduction

Time is money and is now a universal value. Therefore, how to complete a task with a high efficiency is one of the factors that must be considered when choosing a solution to the problem. When there are multiple tasks to be assigned to multiple people working simultaneously, and all the work is expected to be completed in the most efficient way, this is a typical assignment problem [1]. The traditional assignment problem is to minimize the total time (or total cost). In the formation change problem, the goal is to complete the change of the entire formation in the shortest time, that is, to allow the person who takes the most time to complete it in the shortest time.

Path planning is one of the most important issues for motion planning of mobile robots. The goal of path planning is to generate feasible motion paths to enable one or more objects to reach specified targets in a designated area. To move to specified positions in the shortest time, it is very important to find the shortest path for each object, and it is necessary to ensure that this method is effective all the time. For the case of single mobile robot, there are many algorithms can be used. Dijkstra's algorithm [2] is one of the most popular one. However, in the case of multiple objects and multiple targets, it is often impossible to achieve the shortest path for all objects at the same time, so there may be two criterions: the shortest of path summation and the shortest of longest path. The former is a kind of the traditional assignment problem, while the formation change problem is the later one.

There are usually two types of motion planning for multi-robots: one is centralized path planning, and the other is decentralized path planning. The former has a command center that makes overall planning and then notifies each robot of the planned path for execution. The latter has no command center. Each robot determines its own movement path through communication with each other and its own judgment. Since the problem considered in this paper is a formation transformation problem, It would be more appropriate for the robot's movement planning to be mandatory by the command center. Every robot should be noticed where he should move to, and the actual moving path could be determined by himself. We would like to develop an effective algorithm to find an efficient assignment way to achieve this goal.

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2. Preliminary

2.1. Problem description

Suppose that there are n robots on a two-dimensional grid plane. The robots can move laterally or vertically on the plane in the same speed. They are asked to have their formation change in the fastest way. In order to complete this task, we have to minimize the longest route assignment among all robots.

2.2. Distance between two points

The distance between two points on a Cartesian coordinate plane could be determined by the L1-norm of these two points, that is, the sum of the projection of the line segment with these two points as the end points on the axes. For example, suppose the two points are at (x_1, y_1) and (x_2, y_2), the distance between these two points is $|x_1 - x_2| + |y_1 - y_2|$.

2.3. Singularity of a matrix

A square matrix is singular if its determinant is zero. For a possible solution of the considered transformation problem, each robot would be assigned a different target position. The relation of the initial positions of the robots and the possible target position can form a square matrix. The relation of an unassigned positions for a robot can be assigned as 0. For a successful assignment, there will be n non-zero values in the matrix on different rows and columns, forming a square matrix of full rank, i.e., the matrix is nonsingular.

2.4. Munkres assignment algorithm

To solve an assignment problem, Munkres assignment algorithm [3], an improved version of Hungarian algorithm [4], is a useful tool for optimal solution. For a problem with ***n*** workers and ***n*** tasks, and given a cost matrix with element of ***i-th*** row and ***j-th*** column being the cost or benefit for assigning the ***i-th*** worker to do the ***j-th*** task, either the Munkres algorithm or the Hungarian algorithm can find the best assignment way that maximize the total benefit or minimize the total cost for the worker with the rule one-worker-one task.

2.5. Monte Carlo method

The Monte Carlo method is a powerful numerical method for solving a deterministic problem based on random sampling and statistical simulation [5]. When the Monte Carlo method is used for solving an assignment problem, randomly assignments are picked again and again and the maximum of the solution would be found in each trial. As the times of sampling increases, more assignment ways are tested. By minimizing the maximum values, an almost optimal allocation can be obtained.

3. The proposed algorithm

3.1. Deviation of the algorithm

In [6], the algorithm proposed by Hsia, Li and Su is to find the optimal assignment way for the robots from the entries of the distance matrix. In this paper, instead of using the distance matrix to determine the assignment way directly, the minimum path is determined from the singularity check of another matrix named judgement matrix generated by the positional relationship of the elements in the distance matrix. The judgement matrix is of the same dimension as the distance matrix but with all entries 0 initially. Then adjust the corresponding entries of the judgement matrix based on the step-by-step search results in the distance matrix, and then check the singularity of the judgement matrix every time to determine whether it is completed, and then use the Munkres assignment algorithm to determine the target for each robot.

3.2. The proposed algorithm

Step 1: Establish the distance matrix ***D*** with the column number as the starting position, the row number as the target position, and the L1-norm from starting points to target points as the entries.

Step 2: Create a null matrix of same dimension as the distance matrix ***D*** and named it as the judgement matrix ***J***.

Step 3: Set the search value ***s*** as the smallest value of the distance matrix.

Step 4: For any position where the element value of matrix ***D*** is equal to ***s***, fill in an increasing prime number at the same position in matrix ***J***. This means all nonzero elements in ***D*** will be different prime numbers. Note that the smallest prime number is 2, and the next 5 prime numbers are 3, 5, 7, 11, 13.

Step 5: If the judgement matrix ***J*** is singular (determinant is 0), increase the search value to the next large number in the distance matrix ***D*** and go back to Step 4. If not, go to the next step.

Step 6: Now the ***s*** is the smallest number of steps for the formation transformation. Replace all elements in the distance matrix ***D*** whose value greater than ***s*** by a sufficiently large number like 9999.

Step 7: Apply Munkres assignment algorithm to the new distance matrix ***D*** to find the best task assignment for all mobile robots.

4. Two examples that the previous algorithm fail on

Two examples are illustrated in this paper. For both examples, the previous algorithm cannot have actually good results.

Given 7 starting points marked with a circle and 7 target points marked with a cross on an 6x6 grid plane. The starting points are at (1,2), (1,3), (1,4), (1,5), (2, 5), (3,5) and (5,5); and the target points are at (1,1), (1,2), (2,1), (2,3), (3,1), (5,3) and (5,4), as shown in Fig. 1. Create the distance matrix $\boldsymbol{D}$ with entries equal to the distances from the starting positions to the relative target positions. We use the listed sequence as the default sequence. For example, (1,2) as the starting point of the first robot, and (3, 1) as the fifth target point. The distance between these two points is 3. Hence the element of first row and fifth column is 3. The resulting distance matrix is:

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 2 & 2 & 3 & 5 & 6 \\ 2 & 1 & 3 & 1 & 4 & 4 & 5 \\ 3 & 2 & 4 & 2 & 5 & 5 & 4 \\ 4 & 3 & 5 & 3 & 6 & 6 & 5 \\ 5 & 4 & 4 & 2 & 5 & 5 & 4 \\ 6 & 5 & 5 & 3 & 4 & 4 & 3 \\ 8 & 7 & 7 & 5 & 6 & 2 & 1 \end{bmatrix}.$$

The judgement matrix with search value 3 is:

$$\boldsymbol{J} = \begin{bmatrix} 3 & 2 & 19 & 23 & 61 & 0 & 0 \\ 13 & 5 & 47 & 7 & 0 & 0 & 0 \\ 41 & 17 & 0 & 29 & 0 & 0 & 0 \\ 0 & 43 & 0 & 53 & 0 & 0 & 0 \\ 0 & 0 & 0 & 31 & 0 & 0 & 0 \\ 0 & 0 & 0 & 59 & 0 & 0 & 67 \\ 0 & 0 & 0 & 0 & 0 & 37 & 11 \end{bmatrix}.$$

The determinant of this judgement matrix is -3.9×10^{11}. Hence the formation transformation can be completed in 3 steps. However, solving this example using the algorithm proposed in [6] will result in 4 steps to complete the formation transformation. This means that the algorithm proposed by [6] cannot guarantee the optimality of the results obtained. The assignment results is summarized in Table 1 and the moving paths are shown in Fig. 2.

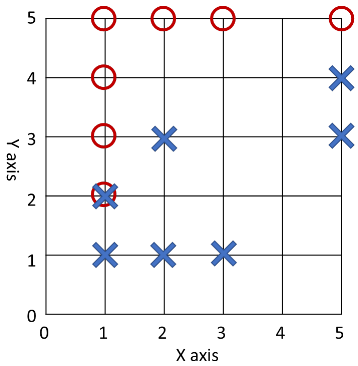

Fig. 1 Formations Example 1

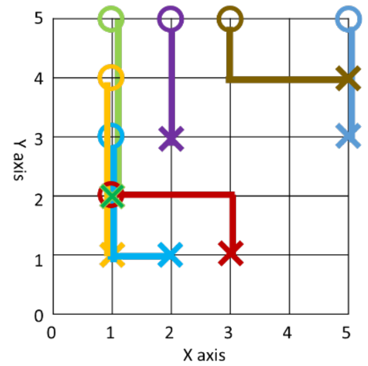

Fig. 2 Assignment result of Example 1

Table 1 The assignment result of Example 1

Starting point	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(2, 5)	(3, 5)	(5, 5)
Target point	(3, 1)	(2, 1)	(1, 1)	(1, 2)	(2, 3)	(5, 4)	(5, 3)
Distance	3	3	3	3	2	3	2

Another example to verify that the new algorithm is better than the previous one is as follows. Given 5 starting points and 5 target points on the same plane, with the starting points at (0,4), (3, 4), (4,1), (5,2), (5,3) and the target points at (0,0), (0,4), (0,5), (1,1), (4,4), as shown in Fig. 3. The distance matrix is:

$$\boldsymbol{D} = \begin{bmatrix} 4 & 0 & 1 & 4 & 4 \\ 7 & 3 & 4 & 5 & 1 \\ 5 & 7 & 8 & 3 & 3 \\ 7 & 7 & 8 & 5 & 3 \\ 8 & 6 & 7 & 6 & 2 \end{bmatrix}.$$

The judgement matrix with search value 5 is:

$$\boldsymbol{J} = \begin{bmatrix} 23 & 2 & 3 & 31 & 37 \\ 0 & 11 & 29 & 43 & 5 \\ 41 & 0 & 0 & 13 & 17 \\ 0 & 0 & 0 & 47 & 19 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}.$$

The determinant of this judgement matrix is 337225. Hence the formation transformation can be completed in 5 steps. However, solving this example using the algorithm proposed in [6] will result in 6 steps in completing the formation transformation. Again, we verified that the algorithm proposed in this paper is better than the previous one. The assignment results are summarized in Table 2.

Table 2 The assignment result of Example 2

Starting point	(0, 4)	(3, 4)	(4, 1)	(5, 2)	(5, 3)
Target point	(0, 5)	(0, 4)	(0, 0)	(1, 1)	(4, 4)
Distance	1	3	5	5	2

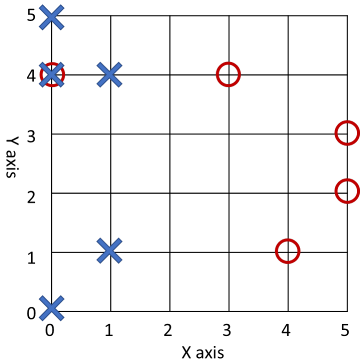

Fig. 3 Formations Example 2

5. Comparison to Monte Carlo method

We conduct a series of comparisons between the proposed algorithm and the Monte Carlo method to verify the superiority of this method. Matlab 2021 is used as the platform for the comparisons. The simulation conditions include formation transformation of 10, 30, 50, 100, 500, and 1000 robots on a grid plane. The staring points and the target positions are all randomly generated for each simulation. Then we use the proposed algorithm to solve the task assignment problem. We also use the Monte Carlo method with the best one out of 10,000 trial for each random data. Compared to the results obtained by both method and find the better one. We have 1000 randomly generated data for each number of robots. This means we have made 60,000 comparisons for different cases. The assignment results via Monte Carlo method could never be better than those via the proposed algorithm. This means that the proposed algorithm is definitely better than Monte Carlo method for the formation transformation problem. The more important point is that the computation time of the proposed method is much smaller than that required by the Monte Carlo method. The comparison results are summarized in Table 3.

Table 3 Comparison-result summary

#- robots	10	30	50	100	500	1000
Count of the results by the new algorithm not worse than those by Monte Carlo method	1000	1000	1000	1000	1000	1000

6. Conclusion

In this paper, we provide a new algorithm for solving the formation transformation problem of multiple mobile robots moving on a 2D grid plane. Each robot can only move horizontally or vertically with the same speed. The requirement is to complete the transformation in the fastest way. The proposed algorithm can be used to systematically find out the best assignment way for each robot the target point so that the robot's formation can be transformed as required in the shortest time. The main difference to the previous algorithms is the introduce of the judgement matrix and the singularity check of this matrix. By comparing with the Monte Carlo method, this algorithm can ensure that the optimal task assignment way could be obtained systematically.

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