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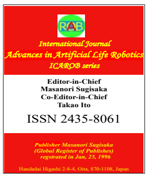

Research Article

Study and circuit design of heterostructured chaotic synchronous control of Chen and Lü systems

Haozhe Sun
College of Electronic Information and Automation, Tianjin University of Science and Technology, 300222, China

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ABSTRACT

In this paper, the Chen system and Lü system are taken as research objects, and the nonlinear feedback synchronization control method is used to synchronize the Chen system with different initial values and the Lü system with different initial values by adding synchronization controllers to achieve heterostructure chaotic synchronization, and the error curves and through the simulation in Simulink, state synchronization curves of the corresponding states in the responding Lü system and the driving Chen system are plotted, which proves that the responding Lü system and the driving Chen system can achieve stable heterostructure chaotic synchronization.. Finally, an analog synchronization control circuit is built in Multisim. By comparing the synchronization curves obtained by simulation and numerical simulation, it is found that the two curves tend to be consistent, so it proves that the synchronization circuit built is correct.

1. Introduction

Chaos is motion that obeys deterministic laws but has randomness. By obeying deterministic laws, it is meant that the motion of the system can be expressed in terms of deterministic dynamical equations; while by motion having randomness, it is meant that chaotic motion has no deterministic trajectory in phase space. The earliest mathematical definition of Chaos was put forward by Li Tianyan and James Alan Yorke in a paper entitled "Period Three Implies Chaos" in 1975. The definition stated that chaos was a special motion state of nonlinear systems. Is a random state appearing in a deterministic system [1].

In this paper, the classical Chen system and the Lü system are taken as the object of study. Analyze their chaotic properties, and based on this, use the method of nonlinear feedback to achieve chaotic synchronization of the two systems [2], and then use the MATLAB software to perform numerical simulation and design the simulated circuits, and perform the circuit simulation in Multisim. The purpose is to provide sufficient theoretical support for the successful application of Chen system and Lü system

heterostructure chaotic synchronization to the field of secure communication in the future.

2. Study and circuit design of heterostructured chaotic synchronous control of Chen and Lü systems

2.1. Modeling of the Drive Chen system

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) \\ \dot{y}_1 = (c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \end{cases} \quad (1)$$

The mathematical model of Chen system is shown in Eq. (1). Where, x_1, y_1, $z_1 \in R$ and $a_1 = 35$, $b_1 = 3$, $c_1 = 28$ is the typical parameter of Chen system. Further analysis from Eq. (1) shows that the Chen system is only symmetric about the z-axis and is dissipative in all cases. Eq. (1) is the mathematical model of the drive Chen system.

Corresponding author's E-mail: 2921407938@qq.com, URL: www.tust.edu.cn

2.2. Modeling of Response Lü System

$$\begin{cases} \dot{x}_2 = a_2(y_2 - x_2) \\ \dot{y}_2 = -x_2 z_2 + c_2 y_2 \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 \end{cases} \quad (2)$$

The mathematical model of Lü system is shown in Eq. (2). Where, x_2, y_2, $z_2 \in R$ and $a_2 = 36$, $b_2 = 3$, $c_2 = 20$ is the typical parameter of Lü system. Further analysis from Eq. (2) shows that the Lü system is only symmetric about the z-axis and is dissipative in all cases.

$$\begin{cases} \dot{x}_2 = a_2(y_2 - x_2) + u_{c1} \\ \dot{y}_2 = -x_2 z_2 + c_2 y_2 + u_{c2} \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 + u_{c3} \end{cases} \quad (3)$$

Eq. (3) is the mathematical model of the response Lü system. $u_c = [u_{c1}\, u_{c2}\, u_{c3}]^T$ is the required synchronization controller. This synchronization controller allows for the synchronization of Chen system and Lü system with completely different structures.

2.3. Design of Synchronization Controller

The mathematical model of the error system can be obtained by making a difference between Eq. (1) and Eq. (3), as shown in Eq. (4).

$$\begin{cases} \dot{e}_1 = \dot{x}_2 - \dot{x}_1 \\ = a_2(y_2 - x_2) - a_1(y_1 - x_1) + u_{c1}, \\ \dot{e}_2 = \dot{y}_2 - \dot{y}_1 \\ = -x_2 z_2 + c_2 y_2 \\ -[(c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1] + u_{c2}, \\ \dot{e}_3 = \dot{z}_2 - \dot{z}_1 \\ = x_2 y_2 - b_2 z_2 - (x_1 y_1 - b_1 z_1) + u_{c3}, \end{cases} \quad (4)$$

In Eq. (4), $e = [e_1 \quad e_2 \quad e_3]^T = [x_2 - x_1 \quad y_2 - y_1 \quad z_2 - z_1]^T$, e_1, e_2, e_3 is the state variable of the error system.

Therefore the synchronization controller is designed as:

$$\begin{cases} u_{c1} = (a_1 - a_2)(y_1 - x_1) - k_1 e_1 \\ u_{c2} = x_2 z_2 - x_1 z_1 + (c_1 - c_2) y_1 \\ +(c_1 - a_1)x_2 - k_2 e_2 \\ u_{c3} = -x_2 y_2 + x_1 y_1 + (b_2 - b_1) z_1 - k_3 e_3 \end{cases} \quad (5)$$

In Eq. (5), k_1, k_2, $k_3 \geq 0$. By substituting Eq. (5) into Eq. (4), the mathematical model of the ultimate error system can be achieved as demonstrated in Eq. (6):

$$\begin{cases} \dot{e}_1 = \dot{x}_2 - \dot{x}_1 = a_2(e_2 - e_1) - k_1 e_1, \\ \dot{e}_2 = \dot{y}_2 - \dot{y}_1 = c_2 e_2 + (c_1 - a_1) e_1 - k_2 e_2, \\ \dot{e}_3 = \dot{z}_2 - \dot{z}_1 = -b_2 e_3 - k_3 e_3, \end{cases} \quad (6)$$

The error system is then written in the form of $\dot{e} = f(e, t)$. It is merely necessary to guarantee that the equilibrium state of the error system at the origin is uniformly asymptotically stable within a wide range, which indicates the achievement of synchronization between the Chen system and the Lü system with distinct structures.

According to Lyapunov's second method [3], as long as $V(\tilde{e})$ is positively definite and $\dot{V}(\tilde{e})$ is negatively definite, the error system is uniformly asymptotically stable at the origin in a large range. This is used to determine the value of parameter $k = [k_1 \quad k_2 \quad k_3]^T$ of the synchronization controller.

Firstly, take the positive definite Lyapunov function $V(\tilde{e}) = 1/2[(a_1 - c_1)e_1^2/a_2 + e_2^2 + e_3^2]$, and then take its derivative:

$$\begin{aligned} \dot{V}(\tilde{e}) &= \frac{a_1 - c_1}{a_2} e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 \\ &= (a_1 - c_1)e_1 e_2 - (a_1 - c_1)e_1^2 - \frac{k_1(a_1 - c_1)}{a_2} e_1^2 \\ &\quad + c_2 e_2^2 + (c_1 - a_1)e_1 e_2 - k_2 e_2^2 - b_2 e_3^2 - k_3 e_3^2 \\ &= -\left[\frac{k_1(a_1 - c_1)}{a_2} + (a_1 - c_1)\right] e_1^2 - (k_2 - c_2)e_2^2 - (k_3 + b_2)e_3^2, \end{aligned} \quad (7)$$

In this case, the error system variable coefficients should satisfy the inequality group: Eq. (8)

$$\begin{cases} -[\frac{k_1(a_1 - c_1)}{a_2} + (a_1 - c_1)] < 0 \\ -(k_2 - c_2) < 0 \\ -(k_3 + b_2) < 0 \end{cases} \quad (8)$$

Solve the set of inequalities to obtain the following result:

$$\begin{cases} k_1 > -a_2 = -36 \\ k_2 > c_2 = 20 \\ k_3 > -b_2 = -3 \end{cases} \quad (9)$$

The value of k satisfying the condition can be arbitrarily chosen. In this paper, $(k_1, k_2, k_3) = (0,25,0)$, then the synchronization controller u_c is as follows:

$$\begin{cases} u_{c1} = -(y_1 - x_1) \\ u_{c2} = x_2 z_2 - x_1 z_1 \\ +33y_1 - 7x_2 - 25y_2 \\ u_{c3} = -x_2 y_2 + x_1 y_1 \end{cases} \quad (10)$$

So far, the mathematical model of synchronous controller has been designed.

The numerical simulation model of the synchronization controller satisfying the condition (Eq. (10)) is built by Simulink in Matlab, as shown in Fig.1. The corresponding state error curve obtained by running is shown in Fig.2, and the corresponding state synchronization curve is shown in Fig.3.

Through observation, it is found that the state error curve gradually tends to 0 with the evolution of time, while the synchronization curve of the corresponding variable continues to run according to the orbit of the Chen drive system after the coincidence. It can be shown that the above formula derivation is correct. Chen system and Lü system can realize different structure chaos synchronization, and Chen system and Lü system can realize the synchronization of different structure chaos. Therefore, the following circuit can be designed and simulated according to the synchronous controller designed above.

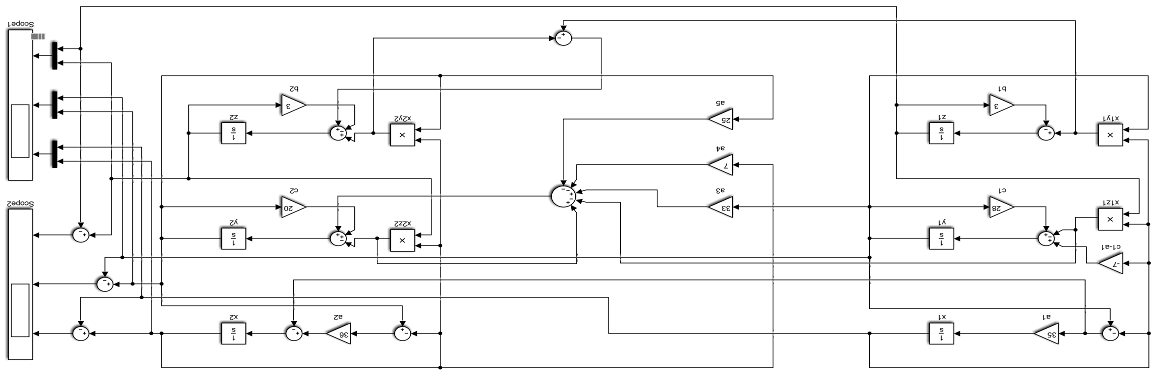

Fig. 1 Numerical simulation model

Fig. 2 State error curve: (a) e_1; (b) e_2; (c) e_3

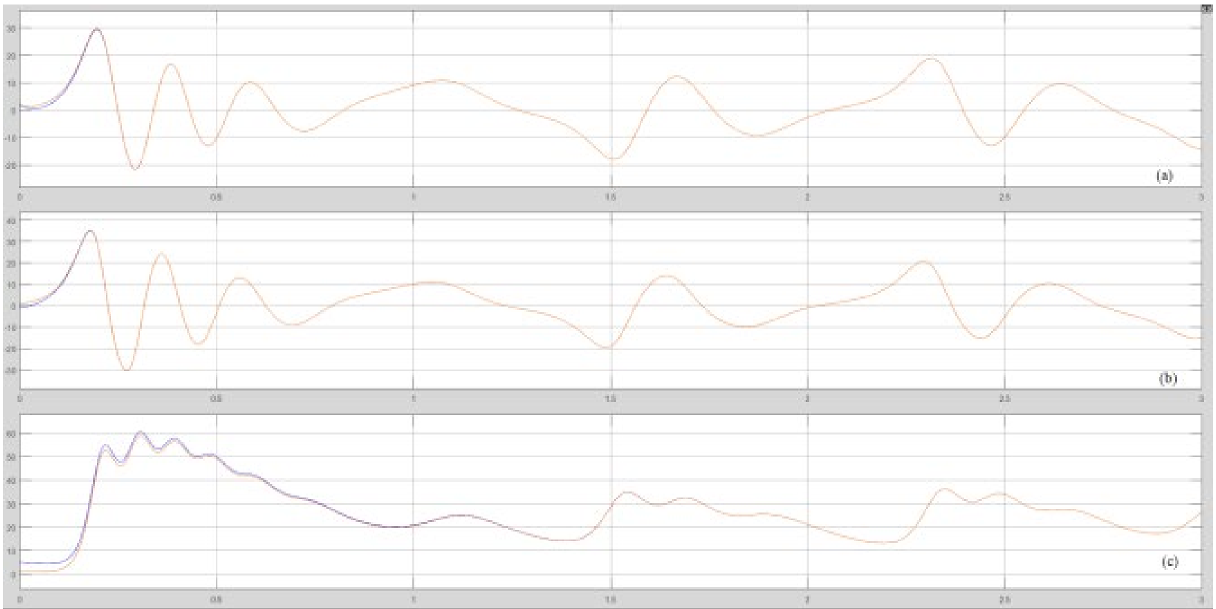

Fig. 3 State synchronization curve: （a）x1-x2；（b）y1-y2；（c）z1-z2

2.4. ***Analog circuit model design and construction***

In the following, the analog circuit model of the synchronous controller designed above is established. To make the designed circuit explicit and manageable, it is determined to adopt the method of improved modular circuit design for the synchronous controller circuit. Simultaneously, to ensure that the operational amplifier operates within the normal working range as much as possible, it is necessary to implement proportional compression transformation and time scale transformation of the variables.

The state expression of this circuit and its derivation are shown in Eq. (11).

$$
\begin{aligned}
\tau_0 u_{c1} &= \tau_0[-(y_1-x_1)] \\
&= -\tau_0 y_1 - \tau_0(-x_1) \\
&= -\frac{1}{R_{14}C_4} y_1 - \frac{1}{R_{15}C_4}(-x_1) \\
\tau_0 u_{c2} &= \tau_0[10x_2z_2 - 10x_1z_1 \\
&\quad +33y_1 - 7x_2 - 25y_2] \\
&= -10\tau_0(-x_2z_2) - 10\tau_0 x_1 z_1 \\
&\quad -33\tau_0(-y_1) - 7\tau_0 x_2 - 25\tau_0 y_2 \\
&= -\frac{1}{10R_{16}C_5}(-x_2z_2) \\
&\quad -\frac{1}{10R_{17}C_5} x_1 z_1 - \frac{1}{R_{13}C_5}(-y_1) \\
&\quad -\frac{1}{R_{19}C_5} x_2 - \frac{1}{R_{20}C_5} y_2 \\
\tau_0 u_{c_3} &= \tau_0(-10x_2y_2 + 10x_1y_1) \\
&= -10\tau_0 x_2 y_2 - 10\tau_0(-x_1y_1) \\
&= -\frac{1}{10R_{21}C_6} x_2 y_2 \\
&\quad -\frac{1}{10R_{22}C_6}(-x_1y_1)
\end{aligned}
\qquad (11)
$$

Substitute $\tau_0 = 100$ into Eq. (11) and take $C_4 = C_5 = C_6 = 10\text{nF}$, and the calculation is as follows:

$$
\begin{aligned}
R_{14} &= R_{15} = \frac{1}{\tau_0 C_4} = \frac{1}{100 \times 10 \times 10^{-9}} \\
&= 1\text{M}\Omega \\
R_{18} &= \frac{1}{33\tau_0 C_5} = \frac{1}{33 \times 100 \times 10 \times 10^{-9}} \\
&= 30.3\text{k}\Omega \\
R_{19} &= \frac{1}{7\tau_0 C_5} = \frac{1}{7 \times 100 \times 10 \times 10^{-9}} \\
&= 142.8\text{k}\Omega \\
R_{20} &= \frac{1}{25\tau_0 C_5} = \frac{1}{25 \times 100 \times 10 \times 10^{-9}} \\
&= 40\text{k}\Omega \\
R_{16} &= R_{17} = R_{21} = R_{22} = \frac{1}{100\tau_0 C_5} \\
&= \frac{1}{100 \times 100 \times 10 \times 10^{-9}} = 10\text{k}\Omega
\end{aligned}
\qquad (12)
$$

The circuit model is built according to the parameters obtained above, are presented in Fig.4. After the connection is accomplished, activate the oscilloscopes corresponding to the variables x-y, x-z and y-z respectively, click the "run" button, await the oscilloscopes to display waveforms, and observe the corresponding waveforms. Fig.5 and Fig.6 show the corresponding state synchronization curve and error curve, respectively

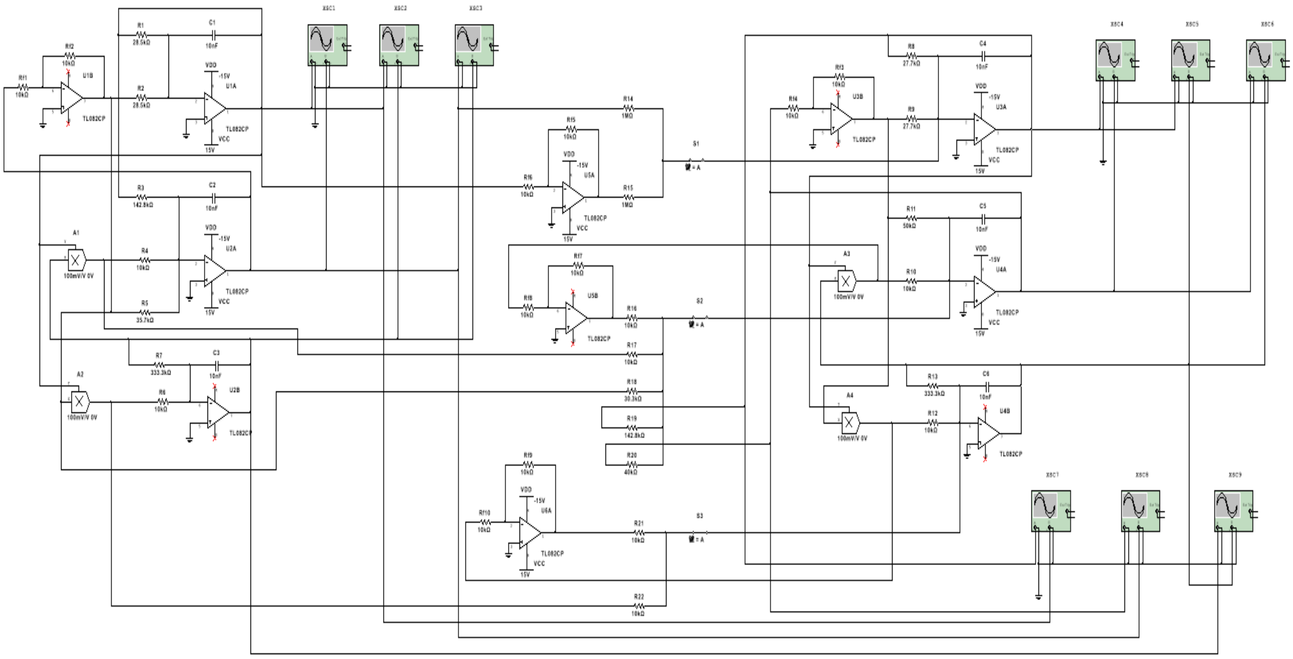

Fig. 4 Circuit simulation model

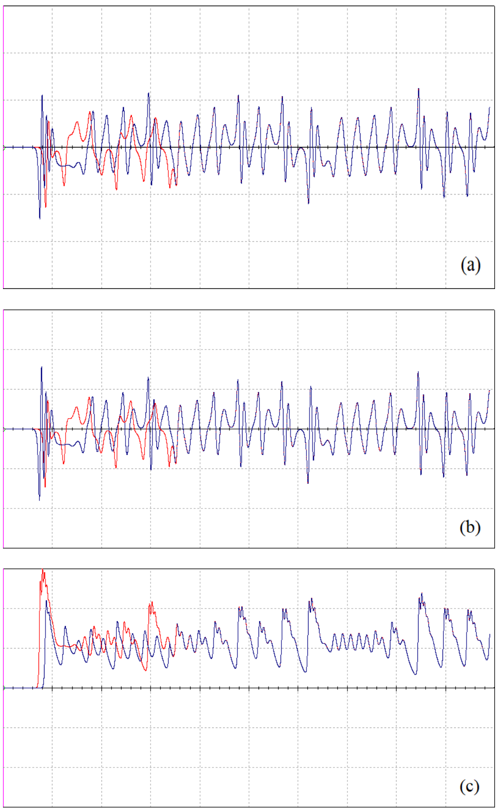

Fig.5. Synchronization state curve: （a） x_1-x_2; （b） y_1-y_2; （c） z_1-z_2

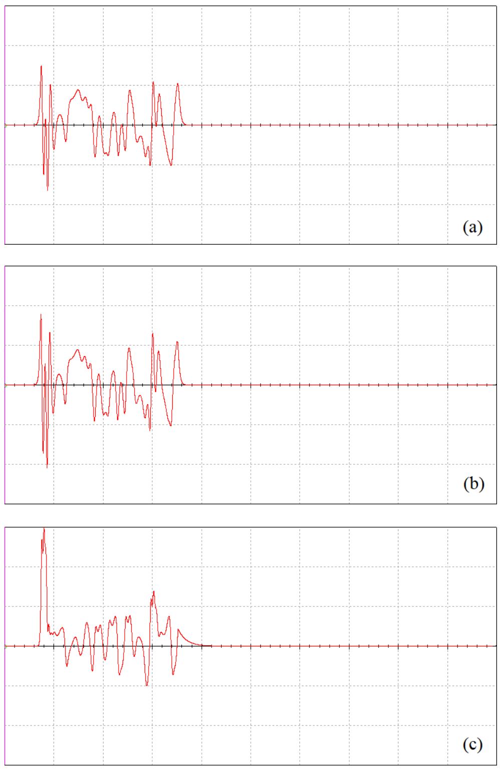

Fig.6. Error curve: （a） e_1 ; （b） e_2 ; （c） e_3

3. Conclusion

This paper begins by separately establishing mathematical models for the driven Chen system and the responsive Lü system. Subsequently, a synchronous controller is designed to eliminate all nonlinear terms, using Lyapunov's second method to derive its parameters.

The designed controller enables numerical simulations in MATLAB, demonstrating that the state error gradually converges to zero over time, while the state synchronization curve follows the trajectory of the Chen drive system.

Then, with the aid of Multisim software, the analog circuit model of Chen system and Lü system with distinct structure chaos synchronization is designed and constructed by the improved modular approach. It constitutes a crucial step in the design of this system to install independent switches for the compensator and the synchronization controller in the system to guarantee separate control, which is largely associated with the realization of the synchronization between the abovementioned two systems. The circuit configured in this manner is more concise and convenient to use than other methods. In the simulation of Multisim software, the oscilloscope indicates that the corresponding state variable curve will reflect diverse waveforms when the independent switch is disconnected and closed. When the switch is disconnected, the observed state curves will oscillate based on their inherent laws. Nevertheless, after the switch is closed, this irregularity will gradually converge to a curve oscillation that follows the operating trajectory of Chen's drive system. The chaotic synchronization analog circuit of the two systems is constructed by employing the improved modular electricity. The nonlinear controller circuit and structure compensator circuit are flexibly connected to the Chen system and the Lü system, and a single switch is utilized to control the access and detachment of the two circuits. When conducting the simulation in Multisim and closing the switch, it can be clearly observed in the oscilloscope that the corresponding state variable curve of the original two does not converge into one but continues to undergo random oscillation, and the oscillation law is in accordance with the variable change mode of the driving Chen's system, thereby verifying the correctness of the improved modular circuit construction. It is also demonstrated that the Chen system and the Lü system achieve chaotic synchronization with different structures.

References

1. Li T, Yorke J A, Period three implies chaos[J]. American mathematical monthly, 1975, 82(10): 985-992.
2. W. Kinzel, A. Englert, I. Kanter. On chaos synchronization and secure Communication[J]. Philosophical Transactions Of The Royal Society A, 2010, 368, 379-389.
3. J. W C .An introduction to Lyapunov's second method[J].Transactions of the American Institute of Electrical Engineers, Part II: Applications and Industry,1962,80(6):325-332.

Authors Introduction

Mr. Haozhe Sun

He is 22 years old, he has obtained a bachelor's degree in engineering and is currently studying for a master's degree in engineering at Tianjin University of Science and Technology. His major is electronic information.